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The nature of free electrons in superfluid helium—a test of quantum mechanics and a basis to review its foundations and make a comparison to classical theory

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Abstract

The Schrödinger equation was originally postulated in 1926 as having a solution of the one electron atom. It gives the principal energy levels of the hydrogen atom as eigenvalues of eigenfunction solutions of the Laguerre differential equation. But, as the principal quantum number $n \gg 1$, the eigenfunctions become nonsensical. Despite its wide acceptance, on deeper inspection, the Schrödinger solution is plagued with many failings as well as difficulties in terms of a physical interpretation that have caused it to remain controversial since its inception. Only the one electron atom may be solved without approximations, but it fails to predict electron spin and leads to models with nonsensical consequences such as negative energy states of the vacuum, infinities, and negative kinetic energy. In addition to many predictions which simply do not agree with observations, the Schrödinger equation predicts noncausality, nonlocality, spooky actions at a distance or quantum telepathy, perpetual motion, and many internal inconsistencies where contradicting statements have to be taken true simultaneously. Recently, the behavior of free electrons in superfluid helium has again forced the issue of the meaning of the wave function. Electrons form bubbles in superfluid helium which reveal that the electron is real and that a physical interpretation of the wave function is necessary. Furthermore, when irradiated with light of energy of about 0.5 to several electron volts (H.J. Maris, J. Low Temp. Phys. 120 (2000) 173), the electrons carry current at different rates as if they exist with different sizes. It has been proposed that the behavior of free electrons in superfluid helium can be explained in terms of the electron breaking into pieces at superfluid helium temperatures (H.J. Maris, J. Low Temp. Phys. 120 (2000) 173). Yet, the electron has proven to be indivisible even under particle accelerator collisions at 90 GeV (LEPII). The nature of the wave function must now be addressed. It is time for the physical rather than the mathematical nature of the wave function to be determined. A theory of classical quantum mechanics (CQM) was derived from first principles by Mills (The grand unified theory of classical quantum mechanics. January 2000 ed; Cranbury, NJ, 2000, Blacklight Power, Inc., (Distributed by Amazon.com; Posted at www.blacklightpower.com)) that successfully applies physical laws on all scales. Using the classical wave equation with the constraint of nonradiation based on Maxwell's equations, CQM gives closed form physical solutions for the electron in atoms, the free electron, and the free electron in superfluid helium. The prediction of fractional principal quantum energy states of the electron in liquid helium match the photoconductivity and mobility observations without requiring that the electron is divisible.

1. Divisible Electron?—is quantum mechanics fundamentally flawed?

In the 103 years since its discovery, there has been no evidence whatsoever that the electron is divisible. But, in order to explain, the rise in current of free electrons in superfluid helium when irradiated with low energy light and the observation of an unexpected plethora of exotic negative charge carriers in superfluid helium with mobilities greater than that of the normal electron, Maris [1] has proposed that the electron breaks into fractional pieces. One piece acquires all of the charge and the other is neutral. Maris shows that the Schrödinger equation solution of the wave function of

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the 1p state, an excited state, will break into two following the 1s to 1p transition of an electron in superfluid helium. This result is a consequence of the localization of the maximum electron density in the extremes of the dumb-bell shaped 1p orbital with the existence of a node at the center of the orbital. The large differences in time scales of the motion of the electron and the motion of the bubble wall means that the Franck–Condon principle should apply and that the wave function of the electron will deform adiabatically (Born–Oppenheimer principle) at this node to result in electron fission. Following the break, one half of the electron's wave function is trapped in each of the two daughter bubbles. As the wave function is the essence of an electron, the electron splits into two.

Electrons may be trapped in superfluid helium as autonomous electron bubbles interloped between helium atoms that have been excluded from the space occupied by the bubble. The surrounding helium atoms maintain the spherical bubble through van der Waals forces. Superfluid helium is an ideal medium to study individual trapped electrons in much the same way that individual ions may be studied in Penning traps. Both represent an ideal system to test quantum mechanics. Maris and other experimental physicists believe that the data on electrons in liquid helium reveals a fundamental flaw in quantum theory which has caused a furor [3–5]. Electron bubbles in superfluid helium reveal that the electron is real and that a physical interpretation of the wave function is necessary. Physicists have always been content to think of the wave function, the unmeasurable entity which describes quantum systems, as a mathematical device with observable consequences. The time has come for the idea to be grounded in reality. For the electron bubbles in helium, Maris's position is that the size of the bubble is determined by how much of the wave function is trapped inside the bubble. If there is no part of the wave function inside the bubble, the bubble will collapse. This makes the wave function seem to be a tangible object. Theoreticians are going to have to address the question: what is a wave function? Is it a real thing, or just a mathematical convenience? [5].

From the time of its inception, quantum mechanics (QM) has been controversial because its foundations are in conflict with physical laws and are internally inconsistent. Interpretations of quantum mechanics such as hidden variables, multiple worlds, consistency rules, and spontaneous collapse have been put forward in an attempt to base the theory in reality. Unfortunately, many theoreticians ignore the requirement that the wave function must be real and physical in order for it to be considered a valid description of reality. For example, regarding this issue Fuchs and Peres believe [6] "Contrary to those desires, quantum theory does *not* describe physical reality. What it does is provide an algorithm for computing *probabilities* for macroscopic events ("detector ticks") that are the consequences of our experimental interventions. This strict definition of the scope of quantum theory is the only interpretation ever needed, whether by experimenters or theorists".

With Penning traps, it is possible to measure transitions including those with hyperfine levels of electrons of single ions. This case can be experimentally distinguished from statistics over equivalent transitions in many ions. Whether many or one, the transition energies are always identical within the resonant line width. So, *probabilities* have no place in describing atomic energy levels. Moreover, quantum theory is incompatible with probability theory as shown in the Appendix.

The Copenhagen interpretation provides another meaning of quantum mechanics. It asserts that what we observe is all we can know; any speculation about what an electron, photon, atom, or other atomic-sized entity really is or what it is doing when we are not looking is just that—speculation. The postulate of quantum measurement asserts that the process of measuring an observable forces it into a state of reality. In other words, reality is irrelevant until a measurement is made. In the case of electrons in helium, the fallacy with this position is that the "ticks" (migration times of electron bubbles) reveal that the electron is real before a measurement is made. Furthermore, experiments on Ba^+ in a Penning trap discussed in the Appendix demonstrate that the postulate of quantum measurement of quantum mechanics is experimentally disproved. These issues and other such flawed philosophies and interpretations of experiments that arise from quantum mechanics are discussed in the Appendix.

QM gives correlations with experimental data. It does not explain the mechanism for the observed data. But, it should not be surprising that it gives good correlations given that the constraints of internal consistency and conformance to physical laws are removed for a wave equation with an infinite number of solutions wherein the solutions may be formulated as an infinite series of eigenfunctions with variable parameters. There are no physical constraints on the parameters. They may even correspond to unobservables such as virtual particles, hyperdimensions, effective nuclear charge, polarization of the vacuum, worm holes, spooky action at a distance, infinities, parallel universes, faster than light travel, etc. If you invoke the constraints of internal consistency and conformance to physical laws, quantum mechanics has never successfully solved a physical problem.

Throughout the history of quantum theory; wherever there was an advance to a new application, it was necessary to repeat a trial-and-error experimentation to find which method of calculation gave the right answers. Often the textbooks present only the successful procedure as if it followed from first principles; and do not mention the actual method by which it was found. In electromagnetic theory based on Maxwell's equations, one deduces the computational algorithm from the general principles. In quantum theory, the logic is just the opposite. One chooses the principle to fit the empirically successful algorithm. For example, we know that it required a great deal of art and

tact over decades of effort to get correct predictions out of Quantum Electrodynamics (QED). For the right experimental numbers to emerge, one must do the calculation (i.e. subtract off the infinities) in one particular way and not in some other way that appears in principle equally valid. There is a corollary, noted by Kallen: from an inconsistent theory, any result may be derived.

Reanalysis of old experiments and many new experiments including electrons in superfluid helium challenge the Schrödinger equation predictions. These issues are discussed in context of a theory of classical quantum mechanics (CQM) derived from first principles by Mills [2]. Using the classical wave equation with the constraint of nonradiation based on Maxwell's equations, CQM gives closed form physical solutions for the electron in atoms, the free electron, and the free electron in superfluid helium which match the observations without requiring that the electron is divisible.

1.1. The Schrödinger theory of the hydrogen atom

In 1911, Rutherford proposed a planetary model for the atom where the electrons revolved about the nucleus (which contained the protons) in various orbits to explain the spectral lines of atomic hydrogen. There was, however, a fundamental conflict with this model and the prevailing classical physics. According to classical electromagnetic theory, an accelerated particle radiates energy (as electromagnetic waves). Thus, an electron in a Rutherford orbit, circulating at constant speed but with a continually changing direction of its velocity vector is being accelerated; thus, the electron should constantly lose energy by radiating and spiral into the nucleus.

An explanation was provided by Bohr in 1913, when he assumed that the energy levels were quantized and the electron was constrained to move in only one of a number of allowed states. Niels Bohr's theory for atomic hydrogen was based on an unprecedented postulate of stable circular orbits that do not radiate. Although no explanation was offered for the existence of stability for these orbits, the results gave energy levels in agreement with Rydberg's equation. Bohr's solution is trivial in that he specified a circular bound orbit which determined that the eccentricity was zero, and he specified the angular momentum as a integer multiple of Planck's constant bar. The solution given by Mills [7] in CGS units is

$$E = -\frac{1}{2}\frac{me^4}{n^2\hbar^2} = -\frac{e^2}{2n^2 a_0} \tag{1}$$

In 1923, de Broglie suggested that the motion of an electron has a wave aspect—$\lambda = h/p$. This was confirmed by Davisson and Germer in 1927 by observing diffraction effects when electrons were reflected from metals. Schrödinger reasoned that if electrons have wave properties, there must be a wave equation that governs their motion. And, in 1926, he proposed the time-independent Schrödinger equation

$$H\Psi = E\Psi \tag{2}$$

where Ψ is the wave function, H is the wave operator, and E is the energy of the wave. To give the sought three quantum numbers, the Schrödinger equation solutions are three-dimensional in space and four-dimensional in space-time

$$\left[\nabla^2 - \frac{1}{v^2}\frac{\delta^2}{\delta t^2}\right]\Psi(r,\theta,\phi,t) = 0, \tag{3}$$

where $\Psi(r,\theta,\phi,t)$ according to quantum theory is the probability density function of the electron as described below and the Appendix under Wave Function Solutions of Quantum Mechanics as Probability Waves are Inconsistent with Probability Theory. When the time harmonic function is eliminated [8],

$$-\frac{\hbar^2}{2\mu}\left[\frac{1}{r^2}\frac{\delta}{\delta r}\left(r^2\frac{\delta\Psi}{\delta r}\right) + \frac{1}{r^2\sin\theta}\frac{\delta}{\delta\theta}\left(\sin\theta\frac{\delta\Psi}{\delta\theta}\right)_{r,\phi} + \frac{1}{r^2\sin^2\theta}\left(\frac{\delta^2\Psi}{\delta\phi^2}\right)_{r,\theta}\right] + V(r)\Psi(r,\theta,\phi) = E\Psi(r,\theta,\phi) \tag{4}$$

where the potential energy $V(r)$ in CGS units is

$$V(r) = -\frac{e^2}{r}. \tag{5}$$

The Schrödinger equation (Eq. (4)) can be transformed into a sum comprising a part that depends only on the radius and a part that is a function of angle only obtained by separation of variables and linear superposition in spherical coordinates. The general form of the solutions for $\psi(r,\theta,\phi)$ are

$$\psi(r,\theta,\phi) = \sum_{l,m} R_{nlm}(r)Y_{lm}(\theta,\phi) \tag{6}$$

where l and m are separation constants. The azimuthal (θ) part of Eq. (4) is the generalized Legendre equation which is derived from the Laplace equation by Jackson (Eq. (3.9) of Jackson [9]). The solutions for the full angular part of Eq. (4), $Y_{lm}(\theta,\phi)$, are the spherical harmonics

$$Y_{lm}(\theta,\phi) = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}}P_l^m(\cos\theta)e^{im\phi}. \tag{7}$$

By substitution of the eigenvalues corresponding to the angular part [8, pp. 221–224] the Schrödinger equation becomes the radial equation, $R(r)$, given by

$$-\frac{\hbar^2}{2mr^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \left[\frac{\hbar^2 l(l+1)}{2mr^2} + V(r)\right]R(r) = ER(r). \tag{8}$$

The time-independent Schrödinger equation is similar to Eq. (20) of Mills [7], except that the solution is for the distribution of a spatial wave function in three dimensions rather than the dynamical motion of a point particle of mass m along a one-dimensional trajectory. Electron motion is implicit in the Schrödinger equation. For wave propagation in three dimensions, the full time-dependent Schrödinger equation is required; whereas, the classical case contains time

derivatives. The kinetic energy of rotation K_{rot} is given classically by

$$K_{rot} = \frac{1}{2}mr^2\omega^2 \tag{9}$$

where m is the mass of the electron. In the time-independent Schrödinger equation, the kinetic energy of rotation K_{rot} is given by

$$K_{rot} = \frac{\ell(\ell+1)\hbar^2}{2mr^2} \tag{10}$$

where

$$L = \sqrt{\ell(\ell+1)\hbar^2} \tag{11}$$

is the value of the electron angular momentum L for the state $Y_{lm}(\theta,\phi)$.

In the case of the ground state of hydrogen, the Schrödinger equation solution is trivial for an implicit circular bound orbit which determines that the eccentricity is zero, and with the specification that the electron angular momentum is Planck's constant bar. With $k = e^2$, Eq. (25) of Mills [7] in CGS units becomes

$$E = -\frac{1}{2}\frac{me^4}{\hbar^2} = -\frac{e^2}{2a_0} \tag{12}$$

which corresponds to $n = 1$ in Eq. (1). Many problems in classical physics give three quantum numbers when three spatial dimensions are considered. In order to obtain three quantum numbers, the Schrödinger equation requires that the solution is for the distribution of a spatial wave function in three dimensions with implicit motion rather than a one-dimensional trajectory of a point particle as shown below. However, this approach gives rise to predictions about the angular momentum and angular energy which are not consistent with experimental observations as well as a host of other problems which are summarized below.

The radial equation may be written as [10]

$$\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \frac{2mr^2}{\hbar^2}\left[E - V(r) - \frac{l(l+1)\hbar^2}{2mr^2}\right]R(r) = 0. \tag{13}$$

Let $U(r) = rR(r)$, then the radial equation reduces to

$$U'' + \frac{2m}{\hbar^2}\left[E - V(r) - \frac{l(l+1)\hbar^2}{2mr^2}\right]U = 0 \tag{14}$$

where

$$\psi = \frac{1}{r}U_{lm}(r)Y_{lm}(\theta,\phi). \tag{15}$$

Substitution of the potential energy given by Eq. (5) into Eq. (14) gives for sufficiently large r

$$U''_{\infty} - \left(\frac{\alpha}{2}\right)^2 U = 0 \tag{16}$$

provided we define

$$\left(\frac{\alpha}{2}\right)^2 = \frac{-2mE}{\hbar^2} \tag{17}$$

where α is the eigenvalue of the eigenfunction solution of the Schrödinger equation given *infra* having units of reciprocal length and E is the energy levels of the hydrogen atom. To arrive at the solution which represents the electron, a suitable boundary condition must be imposed. Schrödinger postulated a boundary condition: $\Psi \to 0$ as $r \to \infty$, which leads to a purely mathematical model of the electron. This equation is *not* based on first principles, has no validity as such, and should not be represented as so. The right-hand side of Eq. (17) must be *postulated* in order that the Rydberg equation is obtained as shown below. The postulate is implicit since Eq. (17) arises from the Schrödinger which is postulated. It could be defined *arbitrarily*, but is justified because it gives the Rydberg formula. That Schrödinger guessed the accepted approach is not surprising since many approaches were contemplated at this time [11], and since none of these approaches were superior, Schrödinger's approach prevailed.

The solution of Eq. (16) that is consistent with the boundary condition is

$$U_{\infty} = c_1 e^{(\alpha/2)r} + c_2 e^{-(\alpha/2)r}. \tag{18}$$

In the case that α is real, the energy of the particle is negative. In this case U_{∞} will not have an integrable square if c_1 fails to vanish wherein the radial integral has the form

$$\int_0^{\infty} R^2 r^2\,dr = \int U_{\infty}^2\,dr. \tag{19}$$

It is shown below that the solution of the Schrödinger corresponds to the case wherein C_1 fails to vanish. Thus, the solutions with sufficiently large r are infinite. The same problem arises in the case of a free electron that is ionized from hydrogen. If α is imaginary, which means that E is positive, Eq. (16) is the equation of a linear harmonic oscillator [12]. U_{∞} shows sinusoidal behavior; thus, the wave function for the free electron cannot be normalized and is infinite. In addition, the angular momentum of the free electron is infinite since it is given by $\ell(\ell+1)\hbar^2$ (Eq. (11)) where $\ell \to \infty$.

In order to solve the bound electron states, let

$$E = -W \tag{20}$$

so that W is positive. In Eq. (13), let $r = x/\alpha$ where α is given by Eq. (17).

$$x\frac{d^2R}{dx^2} + 2\frac{dR}{dx} + \left[\frac{2me^2}{\hbar^2\alpha} - \frac{x}{4} - \frac{l(l+1)}{x}\right]R = 0. \tag{21}$$

Eq. (21) is the differential equation for associated Laguerre functions given in general form by

$$xy'' + 2y' + \left[n^* - \frac{k-1}{2} - \frac{x}{4} - \frac{k^2-1}{4x}\right]y = 0 \tag{22}$$

which has a solution possessing an integrable square of the form

$$y = e^{-x/2}x^{(k-1)/2}L_{n^*}^k(x). \tag{23}$$

provided that n^* and k are positive integers. However, n^* does not have to be an integer, it may be any *arbitrary*

constant β. Then the corresponding solution is [13]

$$y = e^{-x/2}x^{(k-1)/2}\frac{d^k}{dx^k}L_\beta(x). \tag{24}$$

In the case that n^* is chosen to be an integer in order to obtain the Rydberg formula, $n^* - k \geqslant 0$ since otherwise $L^k_{n^*}(x)$ of Eq. (23) would vanish. By comparing Eqs. (21) and (22),

$$\frac{k^2-1}{4} = \ell(\ell+1). \tag{25}$$

Thus,

$$k = 2\ell + 1 \tag{26}$$

and

$$n^* - \frac{k-1}{2} = n^* - \ell = \frac{me^2}{\hbar}\left(\frac{\alpha}{2}\right)^{-1}. \tag{27}$$

Substitution of the value of α and solving for W gives

$$W = \frac{1}{2}\frac{me^4}{(n^*-\ell)^2\hbar^2}. \tag{28}$$

Because of the conditions on n^* and k, the quantity $n^* - \ell$ cannot be zero. It is usually denoted by n and called the principal quantum number. The energy states of the hydrogen atom are

$$W_n = -E_n = \frac{1}{2}\frac{me^4}{n^2\hbar^2} \tag{29}$$

and the corresponding eigenfunctions from Eq. (23) are

$$R_{n,\ell} = c_{n,\ell}e^{-x/2}x^\ell L^{2\ell+1}_{n+\ell}(x) \tag{30}$$

where the variable x is defined by

$$x = \alpha r = \frac{\sqrt{8mW}}{\hbar}r = \frac{2me^2}{n\hbar^2}r. \tag{31}$$

In the Bohr theory of the hydrogen atom, the first orbital has a radius in CGS units given by

$$a_0 = \frac{\hbar^2}{me^2} = 0.53 \times 10^{-8}\ \text{cm}. \tag{32}$$

Thus, $\alpha = 2/na_0$ and

$$x = \frac{2}{n}\frac{r}{a_0}. \tag{33}$$

The energy states of the hydrogen atom in CGS units in terms of the Bohr radius are given by Eq. (1). From Eq. (30), $R_{n,\ell}$ for the hydrogen atom ground state is

$$R_{1,0} = c_{1,0}e^{-r/a_0}L^1_1 = 2a_0^{-3/2}e^{-r/a_0}. \tag{34}$$

For this state

$$Y_{00} = \text{constant} = (4\pi)^{-1/2} \tag{35}$$

when the function is normalized. Thus, the "ground state function" *defined* by the *arbitrary* selection of n^* at Eq. (22) is

$$\psi_0 = (\pi a_0^3)^{-1/2}e^{-r/a_0}. \tag{36}$$

In fact, the Schrödinger can only yield integer eigenvalue solutions by selection from an infinite number of possibilities since the solution is over all space with no boundary (i.e. 0 to ∞). In contrast, wave equation solutions with integers are common for boundary constrained systems such as wave guides and resonators.

1.2. The postulated Schrödinger equation fails to solve the hydrogen atom correctly

The paper by Mills [7] rigorously analyzes the Schrödinger equation. One of many possible solutions of the postulated Schrödinger equation gives the Rydberg levels as does the theory of Bohr. On this basis alone, it is justified despite its inconsistency with physicals laws and numerous experimental observations such as

- The appropriate eigenvalue must be postulated and the variables of the Laguerre differential equation must be defined as integers in order to obtain the Rydberg formula.
- The Schrödinger equation is not Lorentzian invariant.
- The Schrödinger equation violates first principles including special relativity and Maxwell's equations.
- The Schrödinger equation gives no basis why excited states are radiative and the 13.6 eV state is stable. Mathematics does not determine physics. It only models physics.
- In the time-independent Schrödinger equation, the kinetic energy of rotation K_{rot} is given by Eq. (10) where the value of the electron angular momentum L for the state $Y_{lm}(\theta,\phi)$ is given by Eq. (11). The Schrödinger equation solutions, Eqs. (10) and (11), predict that the ground state electron has zero angular energy and zero angular momentum, respectively.
- The Schrödinger equation solution, Eq. (11), predicts that the ionized electron may have infinite angular momentum.
- The Schrödinger equation solutions, Eqs. (10) and (11), predict that the excited state rotational energy levels are nondegenerate as a function of the ℓ quantum number, even in the absence of an applied magnetic field, and the predicted energy is over six orders of magnitude of the observed nondegenerate energy in the presence of a magnetic field. In the absence of a magnetic field, no preferred direction exists. In this case, the ℓ quantum number is a function of the orientation of the atom with respect to an arbitrary coordinate system. Therefore, the nondegeneracy is nonsensical and violates conservation of angular momentum of the photon.
- The Schrödinger equation predicts that each of the functions that corresponds to a highly excited state electron is not integrable and can not be normalized; thus, each is infinite.
- The Schrödinger equation predicts that the ionized electron is sinusoidal over all space and cannot be normalized; thus, it is infinite.
- The Heisenberg uncertainty principle arises as the standard deviation in the electron probability wave, but experimentally it is not the basis of wave particle duality as shown in the Appendix.
- Quantum mechanical textbooks express the movement of the electron, and the Heisenberg uncertainty principle is an expression of the statistical aspects of this movement. McQuarrie [8, back cover], gives the electron speed in the

$n = 1$ state of hydrogen as 2.18764×10^6 m/s. Remarkably, the uncertainty in the electron speed according to the uncertainty principle is 1.4×10^7 m/s [8, p. 38] which is an order of magnitude larger that the speed.

- Experimentally the electron has precise velocity, kinetic energy, and angular momentum. Acquiring these exact properties instantaneously defies all known physical principles.
- The correspondence principle does not hold experimentally.
- The Schrödinger equation does not predict the electron magnetic moment and misses the spin quantum number all together.
- The Schrödinger equation is not a wave equation since it gives the velocity squared proportional to the frequency.
- The Schrödinger equation is not consistent with conservation of energy in an inverse potential field wherein the binding energy is equal to the kinetic energy and the sum of the binding energy and the kinetic energy is equal to the potential energy.
- The Schrödinger equation permits the electron to exist in the nucleus which is a state that is physically nonsensical with infinite potential energy and infinite negative kinetic energy.
- The Schrödinger equation interpreted as a probability wave of a point particle cannot explain neutral scattering of electrons from hydrogen.
- The Schrödinger equation interpreted as a probability wave of a point particle gives rise to infinite magnetic and electric energy in the corresponding fields of the electron.
- A modification of the Schrödinger equation was developed by Dirac to explain spin which relies on the unfounded notions of negative energy states of the vacuum, virtual particles, and gamma factors.

The success of quantum mechanics can be attributed to (1) the lack of rigor and unlimited tolerance to ad hoc assumptions in violation of physical laws, (2) fantastical experimentally immeasurable corrections such as virtual particles, vacuum polarizations, effective nuclear charge, shielding, ionic character, compactified dimensions, and renormalization, and (3) curve fitting parameters that are justified solely on the basis that they force the theory to match the data. Quantum mechanics is now in a state of crisis with constantly modified versions of matter represented as undetectable minuscule vibrating strings that exist in many unobservable hyperdimensions, that can travel back and forth between undetectable interconnected parallel universes. (An analysis of the many failings of quantum mechanics are given in the Appendix.) And, recent data shows that the expansion of the universe is accelerating. This observation has shattered the long held unquestionable doctrine of the origin of the universe as a big bang [14]. It may be time to reconsider the roots of quantum theory, namely the theory of the hydrogen atom. Especially in light of the observation of real electron bubbles in helium which require that the electron is divisible in order for the Schrödinger equation to explain the increase in conductivity upon irradiation with low-energy light. This argument is reinforced by the demonstration that the electron in atoms, the free electron, and the free electron in superfluid helium can be solved physically rather than mathematically in closed form equations from first principles. The predictions match the observations without requiring that the electron is a probability wave or is divisible.

2. A classical approach to quantum mechanics

2.1. Introduction

A theory of classical quantum mechanics (CQM) was derived from first principles by Mills [2] that successfully applies physical laws on all scales. The classical wave equation is solved with the constraint that a bound electron cannot radiate energy. The mathematical formulation for zero radiation based on Maxwell's equation follows from a derivation by Haus [15]. The function that describes the motion of the electron must not possess spacetime Fourier components that are synchronous with waves traveling at the speed of light. CQM gives closed form solutions for the atom including the stability of the $n = 1$ state and the instability of the excited states, the equation of the photon and electron in excited states, the equation of the free electron, and photon which predict the wave particle duality behavior of particles and light. The current and charge density functions of the electron may be directly physically interpreted. For example, spin angular momentum results from the motion of negatively charged mass moving systematically, and the equation for angular momentum, $\mathbf{r} \times \mathbf{p}$, can be applied directly to the wave function (a current density function) that describes the electron. The magnetic moment of a Bohr magneton, Stern Gerlach experiment, g factor, Lamb shift, resonant line width and shape, selection rules, correspondence principle, wave particle duality, excited states, reduced electron mass, rotational energies, and momenta, orbital and spin splitting, spin-orbital coupling, Knight shift, and spin-nuclear coupling are derived in closed form equations based on Maxwell's equations. The calculations agree with experimental observations.

Many great physicists rejected quantum mechanics. Feynman also attempted to use first principles including Maxwell's Equations to discover new physics to replace quantum mechanics [16]. Other great physicists of the 20th century searched. "Einstein [...] insisted [...] that a more detailed, wholly deterministic theory must underlie the vagaries of quantum mechanics [17]". He felt that scientists were misinterpreting the data.

The results of Mill's theory demonstrate that classical physical laws describe reality on all scales. Unlike quantum mechanics which postulates that different laws apply on the

atomic level, the premise of Mill's theory is that a valid theory must comply with *all* of the following:

- theory must be internally consistent even between widely different phenomena,
- Maxwell's equations,
- conservation of matter/energy,
- conservation of linear and angular momentum,
- charge conservation,
- first and second law of thermodynamics,
- Newton's law in the low speed limit; special relativity otherwise,
- general relativity (e.g. Schwarzschild metric)—no cosmological constant; and Newtonian gravitation in the weak field limit (which demands no cosmological constant),
- a vacuum is a vacuum,
- constant maximum of the speed of light in a vacuum,
- 4 dimensional spacetime,
- the only allowed parameters are the measured fundamental constants.

Quantum mechanics is based on engendering the electron with a wave nature as suggested by the Davisson–Germer experiment and fabricating a set of associated postulates and mathematical rules for wave operators. Quantum mechanics is in violation of Maxwell's equations as shown through application of Haus's condition to the Schrödinger wave functions [15]. Nonradiation based on Maxwell's equations is a necessary boundary constraint since nonradiation is observed experimentally. The shortcoming of QM regarding violation of Maxwell's equations and other first principles are further discussed in the Appendix.

2.2. Mills approach to the solution of the electron

Mills solves the electron by a different approach than that used to solve the Schrödinger wave equation. Rather than using a postulated wave equation with time eliminated in terms of the energy of the electron in a Coulomb field and solving the charge wave (Schrödinger interpretation) or the probability wave (Born interpretation), the solution for the scalar (charge) and vector potential (current) functions of the electron are sought based on first principles. Mills first assumes that the functions that physically describe the mass and charge of the electron in space and time obey the wave equation since it conserves energy and angular momentum. The solution is generalized to be three-dimensional plus time. Rather than use the postulated Schrödinger boundary condition: $\Psi \rightarrow 0$ as $r \rightarrow \infty$, which leads to a purely mathematical model of the electron, the constraint is based on experimental observation that the moving charge must not radiate. Application of the Haus condition based on Maxwell's equations to a generalized three dimension plus time wave equation requires that the functions must be solutions of Eq. (51), a two-dimensional wave equation plus time. This is consistent with first principle laws and ultimately matches experimentation. However, it is unconventional.

The two-dimensional wave equation plus time is given by McQuarrie [8, p. 207]. The electron is confined to two dimensions (θ and ϕ) plus time. Spherical harmonic functions and time harmonic functions are well-known solutions of the angular and time components of the two-dimensional wave equation plus time, respectively. The solutions appear in McQuarrie [8, pp. 206–225]. A constant current function is also a solution of the wave equation. A constant function corresponding to the electron spin function is added to each of the spherical harmonic functions to give the charge (mass) density functions of the electron as a function of time. The integral of a spherical harmonic function over θ and ϕ is zero. The integral of the constant function over θ and ϕ is the total charge (mass) of the electron. These functions comprise the well-known s, p, d, f, etc. electrons or orbitals. In the case that such an electron state arises as an excited state by photon absorption, it is radiative due to a radial dipole term in its current density function since it possesses spacetime Fourier components synchronous with waves traveling at the speed of light.

The excited states are solved, including the radii of the orbitspheres, using Maxwell's equations with the traditional source current boundary constraints at the electron. Quantization arises from the equation of the photon and the electron—not from the solution of the electron alone. After all, each solution is for an excited state created by the absorption of a photon. The solutions are analogous to those of excited resonator modes except that the cavity is dynamic. The field lines from the proton end on the current density function of the electron, and the electric field is zero for $r > r_n$. The trapped photons are a solution of the three-dimensional wave equation plus time given by Maxwell's equations. The electrodynamic field of the photon is a constant function plus a time and spherical harmonic function that is in phase with source currents at the electron which is given by a constant plus a time and spherical harmonic function. Only particular solutions are possible as resonant photons of the electron which is a dynamic resonator cavity. The results are in agreement with first principle physics and experimental observations of the hydrogen atom, excited states, free electron, and free space photon, including the wave particle duality aspects.

2.3. Spin and orbital parameters arise from first principles

An electron is a spinning, two-dimensional spherical surface, called an *electron orbitsphere*, that can exist in a bound state only at specific radii r_n from the nucleus. (See Fig. 1 for a pictorial representation of an orbitsphere.) The result for the $n=1$ state of hydrogen is that the charge density function remains constant with each point on the surface moving at the same angular and linear velocity. The constant function solution of the two-dimensional wave equation corresponds to the spin function which has a corresponding spin angular

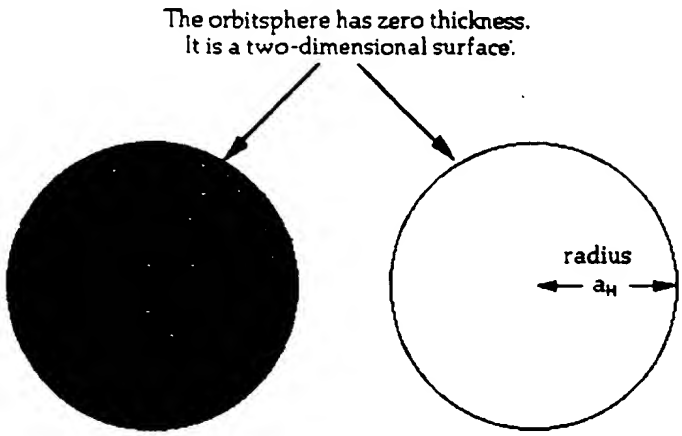

Fig. 1. The orbitsphere is a two-dimensional spherical shell with the Bohr radius of the hydrogen atom.

momentum that may be calculated from $\mathbf{r} \times \mathbf{p}$ applied directly to the current density function that describes the electron. The radius of the nonradiative ($n = 1$) state is solved using the electromagnetic force equations of Maxwell relating the charge and mass density functions, wherein the angular momentum of the electron is given by Planck's constant bar (Eq. (1.165) of [2]). The reduced mass arises naturally from an electrodynamic interaction between the electron and the proton, rather than from a point mass revolving around a point nucleus in the case of Schrödinger wave equation solutions which presents an internal inconsistency since the wave functions are spherically symmetrical.

CQM gives closed form solutions for the resonant photons and excited state electron functions. Angular momentum of the photon given by $\mathbf{m} = (1/8\pi)\mathrm{Re}[\mathbf{r} \times (\mathbf{E} \times \mathbf{B}^*)]\hbar$ is conserved. The change in angular velocity of the electron is equal to the angular frequency of the resonant photon. The energy is given by Planck's equation. The predicted energies, Lamb shift, hyperfine structure, resonant line shape, line width, selection rules, etc. are in agreement with observation.

The radii of excited states are solved using the electromagnetic force equations of Maxwell relating the field from the charge of the proton, the electric field of the photon, and charge and mass density functions of the electron wherein the angular momentum of the electron is given by Planck's constant bar (Eq. (1.165) of [2]).

For excited states of the hydrogen atom, the constant function solution of the two-dimensional wave equation corresponds to the spin function. Each spherical harmonic function modulates the constant spin function and corresponds to an orbital function of a specific excited state with a corresponding phased-matched-trapped photon and orbital angular momentum. Thus, the spherical harmonic function behaves as a charge density wave which travels time harmonically on the surface of the orbitsphere about a specific axis. (See Fig. 2 for a pictorial representation.) An amplitude of the corresponding orbital energy may be calculated from Maxwell's equations. Since the constant function is modulated harmonically, the time average of the orbital energy is zero except in the presence of a magnetic field. Nondegeneracy of energy levels arises from spin, orbital, and spin-orbital coupling interactions with the applied field. The electrodynamics interaction with the magnetic field gives rise to the observed hyperfine splitting of the hydrogen spectrum.

Many inconsistencies arise in the case of the corresponding solutions of the Schrödinger wave equation. For example, where is the photon in excited states given by the Schrödinger equation? And a paradox arises for the change in angular momentum due to photon absorption. The Schrödinger equation solutions for the kinetic energy of rotation K_{rot} is given by Eq. (10) and the value of the electron angular momentum L for the state $Y_{lm}(\theta, \phi)$ given by Eq. (11) predict that the excited state rotational energy levels are nondegenerate as a function of the ℓ quantum number even in the absence of an applied magnetic field, and the predicted energy is over six orders of magnitude of the observed nondegenerate energy in the presence of a magnetic field. In the absence of a magnetic field, no preferred direction exists. In this case, ℓ the quantum number is a function of the orientation of the atom with respect to an arbitrary coordinate system. Therefore, the nondegeneracy is nonsensical and violates conservation of angular momentum of the photon.

In quantum mechanics, the spin angular momentum of the electron is called the "intrinsic angular momentum" since no physical interpretation exists. The Schrödinger equation is not Lorentzian invariant in violation of special relativity. It failed to predict the results of the Stern–Gerlach experiment which indicated the need for an additional quantum number. Quantum electrodynamics was proposed by Dirac in 1926

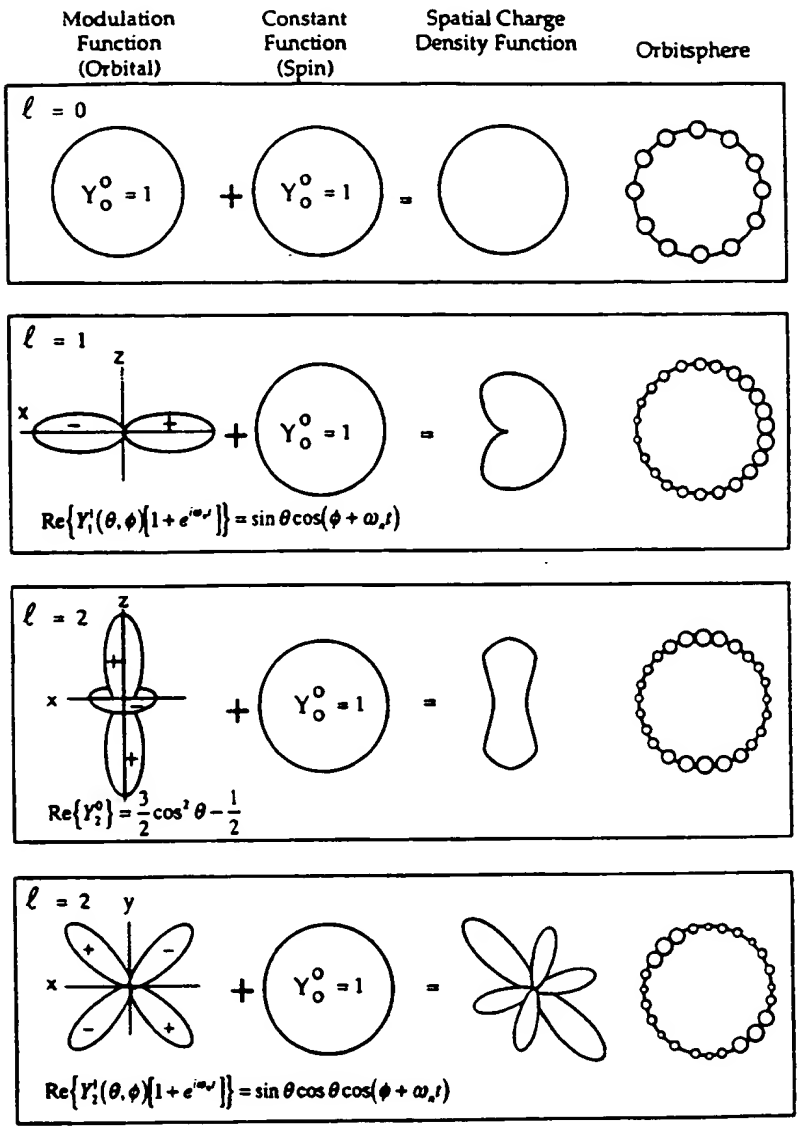

Fig. 2. The orbital function modulates the constant (spin) function (shown for $t = 0$; cross-sectional view).

to provide a generalization of quantum mechanics for high energies in conformity with the theory of special relativity and to provide a consistent treatment of the interaction of matter with radiation. It is fatally flawed. From Weisskopf [18], "Dirac's quantum electrodynamics gave a more consistent derivation of the results of the correspondence principle, but it also brought about a number of new and serious difficulties." Quantum electrodynamics: (1) *does not explain nonradiation of bound electrons*; (2) contains an internal inconsistency with special relativity regarding the classical electron radius—the electron mass corresponding to its electric energy is infinite (the Schrödinger equation fails to predict the classical electron radius); (3) it admits solutions of negative rest mass and negative kinetic energy; (4) the interaction of the electron with the predicted zero-point field fluctuations leads to infinite kinetic energy and infinite electron mass; and (5) Dirac used the unacceptable states of negative mass for the description of the vacuum; yet, infinities still arise. Dirac's equation which was postulated to explain spin relies on the unfounded notions of negative energy states of the vacuum, virtual particles, and gamma factors. All of these features are untenable or are inconsistent

with observation. These problems regarding spin and orbital angular momentum and energies and the classical electron radius are nonexistent with CQM solutions [2].

Furthermore, Mills [2, pp. 1–21] shows that the Schrödinger equation may be transformed into a form consistent with first principles. In the case that the potential energy of the Hamiltonian, H, is a constant times the wave number, the Schrödinger equation is the well-known Bessel equation. Then with one of the solutions for the wave function Ψ (a current density function rather than a probability wave) is equivalent to an inverse Fourier transform. According to the duality and scale change properties of Fourier transforms, the energy equation of Mills theory and that of quantum mechanics are identical, the energy of a radial Dirac delta function of radius equal to an integer multiple of the radius of the hydrogen atom.

2.4. *The Mills theory—a classical quantum theory*

One-electron atoms include the hydrogen atom, He^+, Li^{2+}, Be^{3+}, and so on. The mass-energy and angular momentum of the electron are constant; this requires that the equation of motion of the electron be temporally and spatially harmonic. Thus, the classical wave equation applies and

$$\left[\nabla^2 - \frac{1}{v^2}\frac{\delta^2}{\delta t^2}\right]\rho(r,\theta,\phi,t) = 0 \tag{37}$$

where $\rho(r,\theta,\phi,t)$ is the charge density function of the electron in time and space. In general, the wave equation has an infinite number of solutions. To arrive at the solution which represents the electron, a suitable boundary condition must be imposed. It is well known from experiments that each single atomic electron of a given isotope radiates to the same stable state. Thus, Mills chose the physical boundary condition of nonradiation of the bound electron to be imposed on the solution of the wave equation for the charge density function of the electron. The condition for radiation by a moving point charge, given by Haus [15], is that its spacetime Fourier transform does possess components that are synchronous with waves traveling at the speed of light. Conversely, it is proposed that the condition for nonradiation by an ensemble of moving point charges that comprises a charge density function is:

> For nonradiative states, the current-density function must *not* possess spacetime Fourier components that are synchronous with waves traveling at the speed of light.

The Haus derivation applies to a moving charge-density function as well, because charge obeys superposition. The Haus derivation is summarized below.

The Fourier components of the current produced by the moving charge are derived. The electric field is found from the vector equation in Fourier space ($\mathbf{k}$, w-space). The inverse Fourier transform is carried over the magnitude of $\mathbf{k}$. The resulting expression demonstrates that the radiation field is proportional to $\mathbf{J}_\perp((\omega/c)\mathbf{n},\omega)$, where $\mathbf{J}_\perp(\mathbf{k},\omega)$ is the spacetime Fourier transform of the current perpendicular to $\mathbf{k}$ and $\mathbf{n} \equiv \mathbf{k}/|k|$. Specifically,

$$\mathbf{E}_\perp(\mathbf{r},\omega)\frac{d\omega}{2\pi} = \frac{c}{2\pi}\int \rho(\omega,\Omega)\,d\omega\,d\Omega\sqrt{\frac{\mu_0}{\varepsilon_0}}\mathbf{n} \times \left(\mathbf{n}\times\mathbf{J}_\perp\left(\frac{\omega}{c}\mathbf{n},\omega\right)e^{i(\omega/c)\mathbf{n}\cdot\mathbf{r}}\right). \tag{38}$$

The field $\mathbf{E}_\perp(\mathbf{r},\omega)d\omega/2\pi$ is proportional to $\mathbf{J}_\perp((\omega/c)\mathbf{n},\omega)$, namely, the Fourier component for which $k=\omega/c$. Factors of ω that multiply the Fourier component of the current are due to the density of modes per unit volume and unit solid angle. An unaccelerated charge does not radiate in free space, not because it experiences no acceleration, but because it has no Fourier component $\mathbf{J}_\perp((\omega/c)\mathbf{n},\omega)$.

The time, radial, and angular solutions of the wave equation are separable. The motion is time harmonic with frequency ω_n. To be a harmonic solution of the wave equation in spherical coordinates, the angular functions must be spherical harmonic functions. A zero of the spacetime Fourier transform of the product function of two spherical harmonic angular functions, a time harmonic function, and an unknown radial function is sought. The solution for the radial function which satisfies the boundary condition is a delta function

$$f(r) = \frac{1}{r^2}\delta(r - r_n), \tag{39}$$

where $r_n = nr_1$ is an allowed radius. Thus, bound electrons are described by a charge-density (mass-density) function which is the product of a radial delta function ($f(r) = (1/r^2)\delta(r - r_n)$), two angular functions (spherical harmonic functions), and a time harmonic function. Thus, an electron is a spinning, two-dimensional spherical surface, called an *electron orbitsphere*, that can exist in a bound state at only specified distances from the nucleus as shown in Fig. 1. More explicitly, the orbitsphere comprises a two-dimensional spherical shell of moving charge.

The total function that describes the spinning motion of each electron orbitsphere is composed of two functions. One function, the spin function, is spatially uniform over the orbitsphere, spins with a quantized angular velocity, and gives rise to spin angular momentum. The other function, the modulation function, can be spatially uniform—in which case there is no orbital angular momentum and the magnetic moment of the electron orbitsphere is one Bohr magneton—or not spatially uniform—in which case there is orbital angular momentum. The modulation function also rotates with a quantized angular velocity.

The corresponding current pattern of the constant charge function of the orbitsphere corresponding to the spin function comprises an infinite series of correlated orthogonal great circle current loops. The current pattern is generated over the surface by two orthogonal sets of an infinite series of nested rotations of two orthogonal great circle current loops where the coordinate axes rotate with the two orthogonal great circles. Each infinitesimal rotation of the infinite

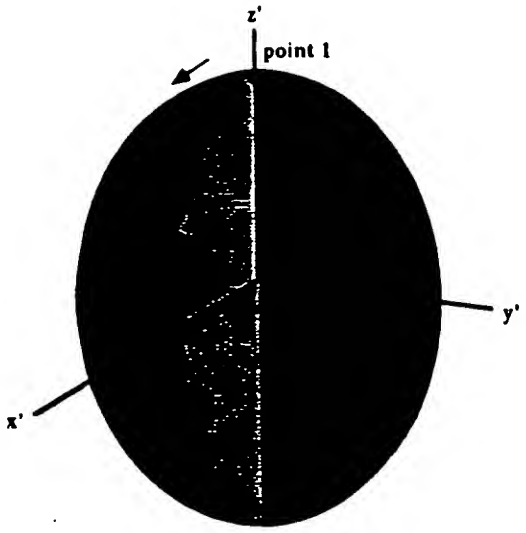

Fig. 3. Two infinitesimal point masses (charges) of two orthogonal great circle current loops in the orbitsphere frame.

series is about the new x-axis and new y-axis which results from the preceding such rotation. For each of the two sets of nested rotations, the angular sum of the rotations about each rotating x-axis and y-axis totals $\sqrt{2}\pi$ radians.

Consider the electron to be evenly distributed within two orthogonal great circle current loops. Then consider two infinitesimal point masses (charges), one and two, of two orthogonal great circle current loops. The Cartesian coordinate system wherein the first current loop lies in the yz-plane, and the second current loop lies in the xz-plane is designated the orbitsphere reference frame. Consider the two point masses, one and two, in the reference frame of the orbitsphere at time zero. Point one is at $x' = 0$, $y' = 0$, and $z' = r_n$ and point two is at $x' = r_n$, $y' = 0$, and $z' = 0$. Let point one move on a great circle toward the negative y'-axis, as shown in Fig. 3, and let point two move on a great circle toward the positive z'-axis, as shown in Fig. 3. The equations of motion, in the reference frame of the orbitsphere are given by

point one:

$$x_1' = 0, \quad y_1' = -r_n \sin(\omega_n t), \quad z_1' = r_n \cos(\omega_n t) \tag{40}$$

point two:

$$x_2' = r_n \cos(\omega_n t), \quad y_2' = 0, \quad z_2' = r_n \sin(\omega_n t). \tag{41}$$

The great circles are rotated by an infinitesimal angle $\Delta\alpha$ (a rotation around the x-axis) and then by $\Delta\alpha$ (a rotation around the new y-axis). The coordinates of each point on the rotated great circle is expressed in terms of the first (x,y,z) coordinates by the following transforms:

point one:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos(\Delta\alpha) & -\sin^2(\Delta\alpha) & -\sin(\Delta\alpha)\cos(\Delta\alpha) \\ 0 & \cos(\Delta\alpha) & -\sin(\Delta\alpha) \\ \sin(\Delta\alpha) & \cos(\Delta\alpha)\sin(\Delta\alpha) & \cos^2(\Delta\alpha) \end{bmatrix} \begin{bmatrix} x_1' \\ y_1' \\ z_1' \end{bmatrix} \tag{42}$$

and $\Delta\alpha' = -\Delta\alpha$ replaces $\Delta\alpha$ for

$$\sum_{n=1}^{\sqrt{2}\pi/\Delta\alpha} \Delta\alpha = \sqrt{2}\pi; \qquad \sum_{n=1}^{\sqrt{2}\pi/|\Delta\alpha'|} |\Delta\alpha'| = \sqrt{2}\pi.$$

point two:

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} \cos(\Delta\alpha) & -\sin^2(\Delta\alpha) & -\sin(\Delta\alpha)\cos(\Delta\alpha) \\ 0 & \cos(\Delta\alpha) & -\sin(\Delta\alpha) \\ \sin(\Delta\alpha) & \cos(\Delta\alpha)\sin(\Delta\alpha) & \cos^2(\Delta\alpha) \end{bmatrix} \begin{bmatrix} x_2' \\ y_2' \\ z_2' \end{bmatrix} \tag{43}$$

and $\Delta\alpha' = -\Delta\alpha$ replaces $\Delta\alpha$ for

$$\sum_{n=1}^{\sqrt{2}\pi/\Delta\alpha} \Delta\alpha = \sqrt{2}\pi; \qquad \sum_{n=1}^{\sqrt{2}\pi/|\Delta\alpha'|} |\Delta\alpha'| = \sqrt{2}\pi.$$

The total orbitsphere is given by reiterations of Eqs. (42) and (43). The output given by the nonprimed coordinates is the input of the next iteration corresponding to each successive nested rotation by the infinitesimal angle where the summation of the rotation about each of the x-axis and the y-axis is

$$\sum_{n=1}^{\sqrt{2}\pi/\Delta\alpha} \Delta\alpha = \sqrt{2}\pi \quad \text{and} \quad \sum_{n=1}^{\sqrt{2}\pi/|\Delta\alpha'|} |\Delta\alpha'| = \sqrt{2}\pi.$$

The current pattern corresponding to point one and point two shown with 8.49 degree increments of the infinitesimal angular variable $\Delta\alpha(\Delta\alpha')$ of Eqs. (42) and (43) is shown from the perspective of looking along the z-axis in Fig. 4. The complete orbitsphere current pattern corresponds to all such correlated points, point one and point two, of the orthogonal great circles shown in Fig. 3 which are rotated according to Eqs. (42) and (43) where $\Delta\alpha(\Delta\alpha')$ approaches zero and the summation of the infinitesimal angular rotations of $\Delta\alpha(\Delta\alpha')$ about the successive x-axes and y-axes is $\sqrt{2}\pi$. The current pattern gives rise to the phenomenon corresponding to the spin quantum number.

The fourth quantum number arises naturally in the Mills theory as derived in the Electron g Factor Section [2]. The Stern–Gerlach experiment implies a magnetic moment of one Bohr magneton and an associated angular momentum quantum number of 1/2. Historically, this quantum number is called the spin quantum number, s ($s = 1/2$; $m_s = \pm 1/2$). Conservation of angular momentum of the orbitsphere permits a discrete change of its "kinetic angular momentum" ($\mathbf{r} \times m\mathbf{V}$) by the field of $\hbar/2$, and concomitantly the "potential angular momentum" ($\mathbf{r} \times e\mathbf{A}$) must change by $-\hbar/2$.

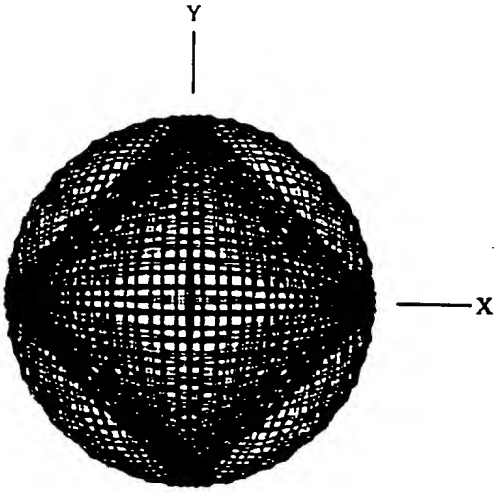

Fig. 4. The current pattern of the orbitsphere shown with 8.49° increments of the infinitesimal angular variable $\Delta\alpha(\Delta\alpha')$ from the perspective of looking along the z-axis.

The flux change, ϕ, of the orbitsphere for $r < r_n$ is determined as follows:

$$\Delta\mathbf{L} = \frac{\hbar}{2} - \mathbf{r} \times e\mathbf{A}, \tag{44}$$

$$= \left(\frac{\hbar}{2} - \frac{e2\pi rA}{2\pi}\right)\hat{z}, \tag{45}$$

$$= \left(\frac{\hbar}{2} - \frac{e\phi}{2\pi}\right)\hat{z}. \tag{46}$$

In order that the change of angular momentum, $\Delta\mathbf{L}$, equals zero, ϕ must be $\Phi_0 = h/2e$, the magnetic flux quantum. Thus, to conserve angular momentum in the presence of an applied magnetic field, the orbitsphere magnetic moment can be parallel or antiparallel to an applied field as observed with the Stern–Gerlach experiment, and the flip between orientations (a rotation of $\pi/2$) is accompanied by the "capture" of the magnetic flux quantum by the orbitsphere. During the spin-flip transition, power must be conserved. Power flow is governed by the Poynting power theorem,

$$\nabla \cdot (E \times H) = -\frac{\partial}{\partial t}\left[\frac{1}{2}\mu_0 H \cdot H\right] - \frac{\partial}{\partial t}\left[\frac{1}{2}\varepsilon_0 E \cdot E\right] - J \cdot E \tag{47}$$

Eq. (48) [2] gives the total energy of the flip transition which is the sum of the energy of reorientation of the magnetic moment (1st term), the magnetic energy (2nd term), the electric energy (3rd term), and the dissipated energy of a fluxon treading the orbitsphere (4th term), respectively.

$$\Delta E_{mag}^{spin} = 2\left(1 + \frac{\alpha}{2\pi} + \frac{2}{3}\alpha^2\left(\frac{\alpha}{2\pi}\right) - \frac{4}{3}\left(\frac{\alpha}{2\pi}\right)^2\right)\mu_B \mathbf{B} \tag{48}$$

$$\Delta E_{mag}^{spin} = g\mu_B \mathbf{B}. \tag{49}$$

The spin–flip transition can be considered as involving a magnetic moment of g times that of a Bohr magneton. The g factor is redesignated the fluxon g factor as opposed to the anomalous g factor. The calculated value of $g/2$ is 1.001 159 652 137. The experimental value of $g/2$ is 1.001 159 652 188(4).

The Mills theory solves the wave equation for the charge density function of the electron. The time, radial, and angular solutions of the wave equation are separable. Also, the radial function for the electron indicates that the electron is two-dimensional. Therefore, the angular mass-density function of the electron, $A(\theta, \phi, t)$, must be a solution of the wave equation in two dimensions (plus time),

$$\left[\nabla^2 - \frac{1}{v^2}\frac{\delta^2}{\delta t^2}\right] A(\theta, \phi, t) = 0, \tag{50}$$

where $\rho(r, \theta, \phi, t) = f(r)A(\theta, \phi, t) = (1/r^2)\delta(r - r_n)A(\theta, \phi, t)$ and $A(\theta, \phi, t) = Y(\theta, \phi)k(t)$

$$\left[\frac{1}{r^2 \sin\theta}\frac{\delta}{\delta\theta}\left(\sin\theta\frac{\delta}{\delta\theta}\right)_{r,\phi} + \frac{1}{r^2 \sin^2\theta}\left(\frac{\delta^2}{\delta\phi^2}\right)_{r,\theta} - \frac{1}{v^2}\frac{\delta^2}{\delta t^2}\right] A(\theta, \phi, t) = 0, \tag{51}$$

where v is the linear velocity of the electron. The charge-density functions including the time-function factor are

$\ell = 0$:

$$\rho(r, \theta, \phi, t) = \frac{e}{8\pi r^2}[\delta(r - r_n)][Y_\ell^m(\theta, \phi) + Y_0^0(\theta, \phi)]. \tag{52}$$

$\ell \neq 0$:

$$\rho(r, \theta, \phi, t) = \frac{e}{4\pi r^2}[\delta(r - r_n)][Y_0^0(\theta, \phi) + \mathrm{Re}\{Y_\ell^m(\theta, \phi)[1 + e^{i\omega_n t}]\}] \tag{53}$$

where $\mathrm{Re}\{Y_\ell^m(\theta, \phi)[1 + e^{i\omega_n t}]\} = \mathrm{Re}[Y_\ell^m(\theta, \phi) + Y_\ell^m(\theta, \phi)e^{i\omega_n t}] = P_\ell^m(\cos\theta)\cos m\phi + P_\ell^m(\cos\theta)\cos(m\phi + \omega_n t)$ and $\omega_n = 0$ for $m = 0$.

The spin function of the electron (see Fig. 1 for the charge function and Fig. 4 for the current function) corresponds to the nonradiative $n = 1$, $\ell = 0$ state of atomic hydrogen which is well known as an s state or orbital. The constant spin function is modulated by a time and spherical harmonic function as given by Eq. (53) and shown in Fig. 2. The modulation or traveling charge density wave corresponds to an orbital angular momentum in addition to a spin angular momentum. These states are typically referred to as p, d, f, etc. orbitals and correspond to an ℓ quantum number not equal to zero. Application of Haus's [18] condition (Eqs. (54)–(56)) also predicts nonradiation for a constant spin function modulated by a time and spherically harmonic orbital function. There is acceleration without radiation. (Also see Abbott and Griffiths and Goedecke [19,20]). However, in the case that such a state arises as an excited state by photon

absorption, it is radiative due to a radial dipole term in its current density function since it possesses spacetime Fourier Transform components synchronous with waves traveling at the speed of light [2].

The Fourier transform of the electron charge density function is a solution of the four-dimensional wave equation in frequency space ($\mathbf{k}$, w-space). Then the corresponding Fourier transform of the current density function $K(s,\Theta,\Phi,\omega)$ is given by multiplying by the constant angular frequency.

$$K(s,\Theta,\Phi,\omega)$$

$$=4\pi\omega_n \frac{\sin(2s_n r_n)}{2s_n r_n} \otimes 2\pi \sum_{v=1}^{\infty} \frac{(-1)^{v-1}(\pi\sin\Theta)^{2(v-1)}}{(v-1)!(v-1)!}$$

$$\frac{\Gamma(\frac{1}{2})\Gamma(v+\frac{1}{2})}{(\pi\cos\Theta)^{2v+1}2^{2v+1}} \frac{2v!}{(v-1)!} s^{-2v}$$

$$\otimes 2\pi \sum_{v=1}^{\infty} \frac{(-1)^{v-1}(\pi\sin\Phi)^{2(v-1)}}{(v-1)!(v-1)!} \frac{\Gamma(\frac{1}{2})\Gamma(v+\frac{1}{2})}{(\pi\cos\Phi)^{2v+1}2^{2v+1}}$$

$$\frac{2v!}{(v-1)!} s^{-2v} \frac{1}{4\pi}[\delta(\omega-\omega_n)+\delta(\omega+\omega_n)]. \quad (54)$$

The motion on the orbitsphere is angular; however, a radial component exists due to special relativistic effects. Consider the radial wave vector of the sinc function. When the radial projection of the velocity is c

$$\mathbf{s}_n \bullet \mathbf{v}_n = \mathbf{s}_n \bullet \mathbf{c} = \omega_n \quad (55)$$

the relativistically corrected wavelength is

$$r_n = \lambda_n \quad (56)$$

(i.e. the lab frame motion in the angular direction goes to zero as the velocity approaches the speed of light). Substitution of Eq. (56) into the sinc function results in the vanishing of the entire Fourier transform of the current-density function. Thus, spacetime harmonics of $\omega_n/c = k$ or $(\omega_n/c)\sqrt{\varepsilon/\varepsilon_0} = k$ for which the Fourier transform of the current-density function is nonzero do not exist. Radiation due to charge motion does not occur in any medium when this boundary condition is met.

The orbitsphere is a resonator cavity which traps photons of discrete frequencies. The radius of an orbitsphere increases with the absorption of electromagnetic energy. The solutions to Maxwell's equations for modes that can be excited in the orbitsphere resonator cavity give rise to four quantum numbers, and the energies of the modes are the experimentally known hydrogen spectrum.

The subscript n is used in Eqs. (39) and (74), the quantization condition, appears in the Excited States of the One Electron Atom (Quantization) Section of Mills [2]. Quantization arises as "allowed" solutions of the wave equation corresponding to a resonance between the electron and a photon.

More explicitly, it is well known that resonator cavities can trap electromagnetic radiation of discrete resonant frequencies. The orbitsphere is a resonator cavity which traps photons of discrete frequencies. Thus, photon absorption occurs as an excitation of a resonator mode. The "trapped photon" is a "standing electromagnetic wave" which actually is a circulating wave that propagates along with each great circle current loop of the orbitsphere. The time-function factor, $k(t)$, for the "standing wave" is identical to the time-function factor of the orbitsphere in order to satisfy the boundary (phase) condition at the orbitsphere surface. Thus, the angular frequency of the "trapped photon" has to be identical to the angular frequency of the electron orbitsphere, ω_n. Furthermore, the phase condition requires that the angular functions of the "trapped photon" have to be identical to the spherical harmonic angular functions of the electron orbitsphere. Combining $k(t)$ with the ϕ-function factor of the spherical harmonic gives $e^{i(m\phi-\omega_n t)}$ for both the electron and the "trapped photon" function. The photon is "glued" to the inner orbitsphere surface and the outer nuclear surface as photon source charge density with a radial electric field.

From the application of the nonradiative boundary condition, the instability of excited states as well as the stability of the "ground" state arise naturally in the Mills theory as derived in Stability of Atoms and Hydrinos Section [2]. In addition to the above known states of hydrogen (Eq. (1), the theory predicts the existence of a previously unknown form of matter: hydrogen atoms and molecules having electrons of lower energy than the conventional "ground" state, called *hydrinos* and *dihydrinos*, respectively, where each energy level corresponds to a fractional quantum number.

The central field of the proton corresponds to integer one charge. Excited states comprise an electron with a trapped photon. In all energy states of hydrogen, the photon has an electric field which superposes with the field of the proton. In the $n = 1$ state, the sum is one, and the sum is zero in the ionized state. In an excited state, the sum is a fraction of one (i.e. between zero and one). Derivations from first principles given by Mills demonstrate that each "allowed" fraction corresponding to an excited state is 1/integer. The relationship between the electric field equation and the "trapped photon" source charge-density function is given by Maxwell's equation in two dimensions.

$$\mathbf{n} \bullet (\mathbf{E}_1 - \mathbf{E}_2) = \sigma/\varepsilon_0 \quad (57)$$

where $\mathbf{n}$ is the radial normal unit vector, $\mathbf{E}_1 = 0$ ($\mathbf{E}_1$ is the electric field outside of the orbitsphere), $\mathbf{E}_2$ is given by the total electric field at $r_n = na_H$, and σ is the surface charge density. The electric field of an excited state is fractional; therefore, the source charge function is fractional. It is well known that fractional charge is not "allowed". The reason is that fractional charge typically corresponds to a radiative current density function. The excited states of the hydrogen atom are examples. They are radiative; consequently, they are not stable. Thus, an excited electron decays to the first nonradiative state corresponding to an integer field, $n=1$ (i.e. a field of integer one times the central field of the proton).

Equally valid from first principles are electronic states where the magnitude of the sum of the electric field of the

photon and the proton central field are an integer greater than one times the central field of the proton. These states are nonradiative. A catalyst can effect a transition between these states via a nonradiative energy transfer. Substantial experimental evidence exists that supports the existence of this novel hydrogen chemistry and its applications [21–55] which was missed entirely due to the erroneous concept of the hydrogen atom "ground state" based on the Schrödinger equation. An analysis of the shortcomings of the Schrödinger equation are given in the Appendix and in a paper by Mills [7]. The success of the classical theory of Mills is demonstrated in a recent presentation and recent publications [53–55].

Furthermore, it is a mistake to take the position that solutions of the *Postulated* Schrödinger equation which correspond to n=integer prohibit the existence of hydrogen atoms having a binding energy corresponding to $n = 1/\text{integer}$. Specifically, it is wrong to take the position that $n = 1/\text{integer}$ values cannot exist according to conventional scientific theories since the Schrödinger equation may be solved for either case with equal validity. However, in neither case does the Schrödinger equation provide a physical basis for their existence or behavior with regard to radiation. For example, the Schrödinger equation does not explain the observation that spontaneous emission of radiation does not occur for the state having a binding energy of 13.6 eV. See the Appendix and Mills [7].

2.5. Schrödinger states below $n = 1$

In Eq. (22), n^* does not have to be an integer, it may be any *arbitrary* constant β. Then the corresponding solution is [13]

$$y = e^{-x/2} x^{(k-1)/2} \frac{d^k}{dx^k} L_\beta(x) \tag{58}$$

where k is a positive integer. By comparing Eqs. (21) and (22),

$$\frac{k^2 - 1}{4} = \ell(\ell + 1). \tag{59}$$

Thus,

$$k = 2\ell + 1 \tag{60}$$

and

$$n^* - \frac{k-1}{2} = n^* - \ell = \frac{me^2}{\hbar}\left(\frac{\alpha}{2}\right)^{-1}. \tag{61}$$

Substitution of the value of α and solving for W gives

$$W = \frac{1}{2}\frac{me^4}{(n^* - \ell)^2\hbar^2}. \tag{62}$$

Because of the conditions on n^* and k, the quantity $n^* - \ell$ cannot be zero. In the case that n^* is given as $n^* = 1/p + \ell$ where p is a positive integer, the condition is satisfied. In this case, the principal quantum number is given as $1/p$ where p is a positive integer. The energy states of the hydrogen atom are

$$W_n = -E_n = \frac{1}{2}\frac{me^4}{(\frac{1}{p})^2\hbar^2} \tag{63}$$

and the corresponding eigenfunctions from Eq. (58) are

$$R_{p,\ell} = c_{p,\ell} e^{-x/2} x^\ell \frac{d^{2\ell+1}}{dx^{2\ell+1}} L_{1/p+\ell}(x) \tag{64}$$

where the variable x is defined by

$$x = \alpha r = \frac{\sqrt{8mW}}{\hbar} r = \frac{2me^2}{\frac{1}{p}\hbar^2} r. \tag{65}$$

In the Bohr theory of the hydrogen atom, the first orbital has a radius in CGS units given by

$$a_0 = \frac{\hbar^2}{me^2} = 0.53 \times 10^{-8} \text{ cm}. \tag{66}$$

Thus, $\alpha = 2p/a_0$ and

$$x = \frac{2pr}{a_0}. \tag{67}$$

The energy states of the hydrogen atom in CGS units in terms of the Bohr radius are given by

$$E = -\frac{1}{2}\frac{me^4}{(\frac{1}{p})^2\hbar^2} = -\frac{e^2}{2(\frac{1}{p})^2 a_0}. \tag{68}$$

Eq. (68) corresponds to hydrogen atoms having a binding energy corresponding to $n = 1/\text{integer}$. It is an equally valid solution of the Schrödinger equation for the energy of the hydrogen atom as Eq. (29) corresponding to hydrogen atoms having a binding energy corresponding to n = integer.

2.6. The electron of atomic hydrogen does not spontaneously emit radiation at the $n = 1$ state, but that does not preclude radiationless processes including formation of molecular hydrogen

The nonradiative state of atomic hydrogen which is historically called the "ground state" forms the basis of the boundary condition of Mills theory [2, pp. 33–109] to solve the wave equation. Mills further predicts [2, pp. 138–175] that certain atoms or ions serve as catalysts to release energy from hydrogen to produce an increased binding energy hydrogen atom called a *hydrino atom* having a binding energy of

$$\text{Binding Energy} = \frac{13.6 \text{ eV}}{n^2} \tag{69}$$

where

$$n = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{p} \tag{70}$$

and p is an integer greater than 1, designated as $H[a_H/p]$ where a_H is the radius of the hydrogen atom. Hydrinos are predicted to form by reacting an ordinary hydrogen atom with a catalyst having a net enthalpy of reaction of about

$$m \times 27.2 \text{ eV} \tag{71}$$

where m is an integer. This catalysis releases energy from the hydrogen atom with a commensurate decrease in size of

the hydrogen atom, $r_n = na_H$. For example, the catalysis of $H(n = 1)$ to $H(n = 1/2)$ releases 40.8 eV, and the hydrogen radius decreases from a_H to $\frac{1}{2}a_H$.

It is taught in textbooks that atomic hydrogen cannot go below the ground state. Atomic hydrogen having an experimental ground state of 13.6 eV can only exist in a vacuum or in isolation, and atomic hydrogen cannot go below this ground state in isolation. However, there is no known composition of matter containing hydrogen in the ground state of 13.6 eV. Atomic hydrogen is a free radical and is very reactive. It may react to form a hydride ion or compositions of matter. It is a chemical intermediate which may be trapped as many chemical intermediates may be by methods such as isolation or cryogenically. A hydrino atom may be considered a chemical intermediate that may be trapped in vacuum or isolation. A hydrino atom may be very reactive to form a hydride ion or a novel composition of matter. Hydrogen at predicted lower energy levels, hydrino atoms, has been identified in the extreme ultraviolet emission spectrum from interstellar medium [7]. In addition, new compositions of matter containing hydrogen at predicted lower energy levels have recently been observed in the laboratory [32,34–52], which energy levels are achieved using the novel catalysts. Spectroscopic experiments confirm the catalysis of hydrogen [21–38].

The excited energy states of atomic hydrogen are also given by Eq. (69) except that

$$n = 1, 2, 3, \ldots . \tag{72}$$

The $n = 1$ state is the "ground" state for "pure" photon transitions (the $n = 1$ state can absorb a photon and go to an excited electronic state, but it cannot release a photon and go to a lower-energy electronic state). However, an electron transition from the ground state to a lower-energy state is possible by a nonradiative energy transfer such as multipole coupling or a resonant collision mechanism. These lower-energy states have fractional quantum numbers, $n = 1/\text{integer}$. Processes that occur without photons and that require collisions are common. For example, the exothermic chemical reaction of H + H to form H_2 does not occur with the emission of a photon. Rather, the reaction requires a collision with a third body, M, to remove the bond energy—$H+H+M \rightarrow H_2+M^*$ [56]. The third body distributes the energy from the exothermic reaction, and the end result is the H_2 molecule and an increase in the temperature of the system. Some commercial phosphors are based on nonradiative energy transfer involving multipole coupling [57]. For example, the strong absorption strength of Sb^{3+} ions along with the efficient nonradiative transfer of excitation from Sb^{3+} to Mn^{2+}, are responsible for the strong manganese luminescence from phosphors containing these ions. Similarly, the $n = 1$ state of hydrogen and the $n = 1/\text{integer}$ states of hydrogen are nonradiative, but a transition between two nonradiative states is possible via a nonradiative energy transfer, say $n = 1$ to $n = 1/2$. In these cases, during the transition the electron couples to another electron transition, electron transfer reaction, or inelastic scattering reaction which can absorb the exact amount of energy that must be removed from the hydrogen atom. Thus, a catalyst provides a net positive enthalpy of reaction of $m \times 27.2$ eV (i.e. it absorbs $m \times 27.2$ eV where m is an integer). Certain atoms or ions serve as catalysts which resonantly accept energy from hydrogen atoms and release the energy to the surroundings to effect electronic transitions to fractional quantum energy levels.

Once formed hydrinos have a binding energy given by Eqs. (69) and (70); thus, they may serve as catalysts which provide a net enthalpy of reaction given by Eq. (71). Also, the simultaneous ionization of two hydrogen atoms may provide a net enthalpy given by Eq. (71). Since the surfaces of stars comprise significant amounts of atomic hydrogen, hydrinos may be formed as a source to interstellar space where further transitions may occur [7].

A number of experimental observations lead to the conclusion that atomic hydrogen can exist in fractional quantum states that are at lower energies than the traditional "ground" ($n = 1$) state. For example, the existence of fractional quantum states of hydrogen atoms explains the spectral observations of the extreme ultraviolet background emission from interstellar space [58], which may characterize dark matter as demonstrated in Table 2 of Mills [7].

Laboratory experiments that confirm the novel hydrogen chemistry include extreme ultraviolet (EUV) spectroscopy [21–33,36–38], plasma formation [21–33,36–38], power generation [22–24,29,55], and analysis of chemical compounds [32,36–52,55]. For example, lines observed by EUV spectroscopy could be assigned to transitions of atomic hydrogen to lower energy levels corresponding to lower energy hydrogen atoms called hydrino atoms and the emission from the excitation of the corresponding hydride ions formed from the hydrino atoms [23]. The chemical interaction of catalysts with hydrogen at temperatures below 1000 K has shown surprising results in terms of the emission of the Lyman and Balmer lines [21–38] and the formation of novel chemical compounds [32,34–52]. An energetic plasma in hydrogen was generated by a catalysis reaction at 1% of the theoretical or prior known voltage requirement and with 1000s of times less power input in a system wherein the plasma reaction is controlled with a weak electric field [23,24,29]. The optically measured output power of gas cells for power supplied to the glow discharge increased by over two orders of magnitude depending on the presence of less than 1% partial pressure of certain of catalysts in hydrogen gas or argon–hydrogen gas mixtures [22]. A hydrogen plasma formed by reacting a catalyst with hydrogen was recorded when there was no electric energy input to the reaction [27,28]. The optically measured output power of gas cells for power supplied to the glow discharge increased by over two orders of magnitude depending on the presence of less than 1% partial pressure of certain of catalysts in hydrogen gas or argon–hydrogen gas mixtures [22]. Continuum state emission of Cs^{2+} and Ar^{2+} at 53.3 nm and 45.6 nm, respectively, with

the absence of the other corresponding Rydberg series of lines from these species confirmed the resonant nonradiative energy transfer of 27.2 eV from atomic hydrogen to atomic cesium or Ar^+. The predicted hydride ion of hydrogen catalysis by either cesium atom or Ar^+ catalyst is the hydride ion $H^-(1/2)$. This ion was observed spectroscopically at 407 nm corresponding to its predicted binding energy of 3.05 eV [21].

2.7. Theory of one electron states (hydrogen atom and electron bubble in superfluid helium)

Recently a new challenge to the fundamental foundations of quantum mechanics has arisen based on experiments of free electrons injected into superfluid helium. In order to explain the increase in conductivity observed when electrons in superfluid helium are irradiated with light, British physicist Humphrey Maris has proposed that the electron breaks into equal sized fragments which he calls "electrinos". According to Maris, this process of division of the electron may continue to such that the electron breaks into two and then the 1/2 electrons may divide into two forming 1/4 electrons, and the process may repeat indefinitely.

Electrons do not break into pieces. It is shown infra that the free electron in superfluid helium is an orbitsphere which can act as a resonator cavity and absorb resonant radiation to form stable nonradiative states of radii $n = 1/\text{integer}$ times that of the radius of the electron without an absorbed photon. The solutions are analogous to the solutions of lower-energy states of hydrogen called hydrinos with principal energy levels given by Eqs. (69)–(70) and radii given by a_H/p where a_H is the radius of the hydrogen atoms and p is an integer.

For the electron of the hydrogen atom which comprises a dynamic spherical resonator cavity, the relationship between an allowed radius and the "photon standing wave" wavelength is

$$2\pi r = n\lambda \tag{73}$$

where n is an integer. Now, the question arises: given that this is a resonator cavity, which nonradiative states are possible where the transition is effected by a "trapped photon"? For the electron orbitsphere, a spherical resonator cavity, the relationship between an allowed radius and the electron wavelength is

$$2\pi(nr_1) = 2\pi r_n = n\lambda_1 = \lambda_n \tag{74}$$

where

$n = 1, 2, 3, 4, \ldots,$

$n = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{p}$ where p is an integer,

λ_1 is the allowed wavelength for $n = 1$,

r_1 is the allowed radius for $n = 1$.

The nonradiative boundary condition from Haus [18] and the relationship between the electron and the photon give the "allowed" hydrogen energy states which are quantized as a function of the parameter n. That is the nonradiative boundary condition and the relationship between an allowed radius and the photon standing wave wavelength Eq. (73) gives rise to Eq. (74), the boundary condition for allowed radii and allowed electron wavelengths as a function of the parameter n. Each value of n corresponds to an allowed transition effected by a resonant photon which excites the transition in the orbitsphere resonator cavity. In addition to the traditional integer values $(1, 2, 3, \ldots,)$ of n, values of fractions are allowed according to Eq. (74) which correspond to transitions with an increase in the central field (charge) and decrease in the radius of the orbitsphere. This occurs, for example, when the orbitsphere transfers energy nonradiatively to a catalyst which resonantly accepts the energy. The electron undergoes a transition to a lower energy nonradiative state. The "Excited states of the one electron atom (quantization)" section of Mills [2] gives the solutions of the excited states of atomic hydrogen and the "Blacklight process" section gives the solutions of lower-energy states.

The photon equation must be a solution of Laplace's equation in spherical coordinates. The "trapped photon" field comprises an electric field which provides force balance and a nonradiative orbitsphere. The solution to this boundary value problem of the radial photon electric field of hydrogen states is given by

$$\mathbf{E}_{r_{photon}n,l,m} = \frac{e(na_H)^{\ell}}{4\pi\varepsilon_0}\frac{1}{r^{(\ell+2)}}\left[- Y_0^0(\theta,\phi) + \frac{1}{n}[Y_0^0(\theta,\phi) + \mathrm{Re}\{Y_{\ell}^{m}(\theta,\phi)[1 + e^{i\omega_n t}]\}]\right]$$

$$\omega_n = 0 \quad \text{for } m = 0 \tag{75}$$

$$\ell = 1, 2, \ldots, n-1,$$

$$m_{\ell} = -\ell, -\ell+1, \ldots, 0, \ldots, +\ell.$$

And, the quantum numbers of the electron are n, ℓ, m_{ℓ}, and $m_s = \pm 1/2$ are the same as the corresponding quantum numbers for excited states. $\mathbf{E}_{r_{total}}$ is the sum of the "trapped photon" and proton electric fields,

$$\mathbf{E}_{r_{total}} = \frac{e}{4\pi\varepsilon_0 r^2} + \frac{e(na_H)^{\ell}}{4\pi\varepsilon_0}\frac{1}{r^{(\ell+2)}}\left[- Y_0^0(\theta,\phi) + \frac{1}{n}[Y_0^0(\theta,\phi) + \mathrm{Re}\{Y_{\ell}^{m}(\theta,\phi)[1 + e^{i\omega_n t}]\}]\right] \quad \omega_n = 0 \text{ for } m = 0. \tag{76}$$

For $r = na_H$ and $m = 0$, the total radial electric field is

$$\mathbf{E}_{r_{total}} = \frac{1}{n}\frac{e}{4\pi\varepsilon_0(na_H)^2}. \tag{77}$$

In Eqs. (75)–(77), the excited states of hydrogen correspond to $n = 1, 2, 3, 4, \ldots$, and the hydrino states correspond to $n = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{p}$ where p is an integer.

The "trapped photon" is a "standing electromagnetic wave" which actually is a circulating wave that propagates

along with each great circle current loop of the electron orbitsphere. The time-function factor, $k(t)$, for the "standing wave" is identical to the time-function factor of the orbitsphere in order to satisfy the boundary (phase) condition at the orbitsphere surface. Thus, the angular frequency of the "trapped photon" has to be identical to the angular frequency of the electron orbitsphere, ω_n. Furthermore, the phase condition requires that the angular functions of the "trapped photon" have to be identical to the spherical harmonic angular functions of the electron orbitsphere. Combining $k(t)$ with the ϕ-function factor of the spherical harmonic gives $e^{i(m\phi-\omega_n t)}$ for both the electron and the "trapped photon" function. The photon is "glued" to the inner orbitsphere surface and the outer nuclear surface as photon source charge density with a radial electric field. The angular functions in phase with the corresponding photon functions are given by Eqs. (52) and (53).

The solution of the "trapped photon" field of electrons in helium is analogous to those for hydrinos except that the $-Y_0^0(\theta,\phi)$ term is not present since the central field of the proton is absent and the nature of the field at the origin is equivalent to the solution of the Poisson equation with a delta function inhomogeneity at the origin [9, pp. 110–113].

$$\mathbf{E}_{r_{photon}n,l,m} = C\frac{e(na)^{\ell}}{4\pi\varepsilon_0}\frac{1}{r^{(\ell+2)}}\left[\frac{1}{n}[Y_0^0(\theta,\phi) + \mathrm{Re}\{Y_{\ell}^{m}(\theta,\phi)[1+e^{i\omega_n t}]\}]\right]$$

$$\omega_n = 0 \quad \text{for } m = 0, \tag{78}$$

$$n = 1, \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{4}, \ldots, \tfrac{1}{p},$$

$$\ell = 1, 2, \ldots, n-1,$$

$$m = -\ell, -\ell+1, \ldots, 0, \ldots, +\ell.$$

In Eq. (78), a is the radius of the electron in helium without an absorbed photon. C is a constant expressed in terms of an equivalent central charge. It is determined by the force balance between the centrifugal force of the electron orbitsphere and the radial force provided by the pressure from the van der Waals force of attraction between helium atoms given infra.

2.8. Stability of "ground", hydrino, and helium states

For the below "ground" (fractional quantum) energy states of the hydrogen atom, σ_{photon}, the two-dimensional surface charge due to the "trapped photon" at the electron orbitsphere, is given by Eqs. (5.13) and (2.11) of Mills [2].

$$\sigma_{photon} = \frac{e}{4\pi(r_n)^2}\left[Y_0^0(\theta,\phi) - \frac{1}{n}[Y_0^0(\theta,\phi) + \mathrm{Re}\{Y_{\ell}^{m}(\theta,\phi)[1+e^{i\omega_n t}]\}]\right]\delta(r-r_n),$$

$$n = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots . \tag{79}$$

And, $\sigma_{electron}$, the two-dimensional surface charge of the electron orbitsphere is

$$\sigma_{electron} = \frac{-e}{4\pi(r_n)^2}[Y_0^0(\theta,\phi) + \mathrm{Re}\{Y_{\ell}^{m}(\theta,\phi)[1+e^{i\omega_n t}]\}]\delta(r-r_n). \tag{80}$$

The superposition of σ_{photon} (Eq. (79)) and $\sigma_{electron}$, (Eq. (80)) where the spherical harmonic functions satisfy the conditions given in the "Angular Function" section of Mills [2] is a radial electric monopole represented by a delta function.

$$\sigma_{photon} + \sigma_{electron} = \frac{-e}{4\pi(r_n)^2}\left[\frac{1}{n}Y_0^0(\theta,\phi) + \left(1+\frac{1}{n}\right)\mathrm{Re}Y_{\ell}^{m}(\theta,\phi)[1+e^{i\omega_n t}]\right]\delta(r-r_n),$$

$$n = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots . \tag{81}$$

As given in the "Spacetime Fourier transform of the electron function" section of Mills [2], the radial delta function does not possess spacetime Fourier components synchronous with waves traveling at the speed of light (Eqs. (54)–(56)). Thus, the below "ground" (fractional quantum) energy states of the hydrogen atom are stable. The "ground" ($n=1$ quantum) energy state is just the first of the nonradiative states of the hydrogen atom; thus, it is the state to which excited states decay.

The speed of light in vacuum c is given by

$$c = 1/\sqrt{\mu_0\varepsilon_0}, \tag{82}$$

where μ_0 is the permeability of free-space and ε_0 is the permittivity of free-space. The wave number is given by

$$k_{vacuum} = \frac{2\pi}{\lambda} = \omega\sqrt{\mu_0\varepsilon_0}. \tag{83}$$

The speed of light in a medium such as superfluid helium v is given by

$$v = 1/\sqrt{\mu_0\varepsilon}, \tag{84}$$

where μ_0 is the permeability of free space and ε is the permittivity of the medium. The wave number is given by

$$k_{medium} = \frac{2\pi}{\lambda} = \omega\sqrt{\mu_0\varepsilon}. \tag{85}$$

The ratio of the wave number in vacuum and the wave number in superfluid helium is given by

$$\frac{k_{helium}}{k_{vacuum}} = \frac{\frac{2\pi}{\lambda_{helium}}}{\frac{2\pi}{\lambda_{vacuum}}} = \frac{\omega\sqrt{\mu_0\varepsilon}}{\omega\sqrt{\mu_0\varepsilon_0}}. \tag{86}$$

The frequency of the photon in free space and in helium at the electron must be the same. Thus,

$$k_{helium} = k_{vacuum}\frac{\varepsilon}{\varepsilon_0}. \tag{87}$$

Since $\varepsilon > \varepsilon_0$, the wave number in helium is greater than the wave number in vacuum. Thus, a photon traveling in liquid helium may excite a mode in an electron bubble which is nonradiative. In this case, spacetime harmonics of $\omega_n/c = k$

or $(\omega_n/c)\sqrt{\varepsilon/\varepsilon_0} = k$ for which the Fourier transform of the current-density function is nonzero do not exist. Radiation due to charge motion does not occur in any medium when this boundary condition is met.

The viscosity of superfluid helium is a function of temperature. The observation of the temperature dependence of photon absorption by electrons in superfluid helium to give rise to an increase in conductivity may be explained on this basis. That is, at 1.7 K, the viscosity is sufficiently close to zero such that the angular current of the electron may propagate without energy loss. Roton scattering dominates over phonon scattering at this temperature and below [59]. In this case, resonant absorption may occur between stable nonradiative states; wherein the forces are central. The two-dimensional surface charge due to the "trapped photon" at the electron orbitsphere of an electron in helium has the same form as that given by Eq. (81). Thus, the states of photons absorbed by electrons in helium are stable under these conditions.

2.9. Conservation of angular momentum

The field is time harmonic which is satisfied by the fields spinning around the z-axis at frequency ω_n in phase with the electron. The relationship between the electric field equation and the "trapped photon" source charge density function is given by Maxwell's equation in two dimensions.

$$\mathbf{n} \bullet (\mathbf{E}_1 - \mathbf{E}_2) = \sigma/\varepsilon_0, \tag{88}$$

where $\mathbf{n}$ is the radial normal unit vector, $\mathbf{E}_1 = 0$ ($\mathbf{E}_1$ is the electric field outside of the orbitsphere), $\mathbf{E}_2$ is given by the total electric field at $r_n = na_H$, and σ is the surface charge density. In order that the radial electric field is always positive and the surface charge density is always negative, a constant function adds to a spherical harmonic function; thus, the spherical harmonic function modulates the constant function. The surface charge density has a phase matched pattern. This provides a central force to balance the centrifugal force of the electron. The magnitude of the radial electric field is that which satisfies the boundary condition of force balance at the allowed radii of electron states.

The time harmonic condition is satisfied by the rotation of the fields in phase with the source currents. The spinning field and the corresponding source current at the electron orbitsphere conserves the angular momentum of the photon. The time-averaged angular-momentum density, $\mathbf{m}$, of the emitted photon is given by Eq. (16.61) of Jackson [9, pp. 739–779]

$$\mathbf{m} = \frac{1}{8\pi}\,\mathrm{Re}[\mathbf{r} \times (\mathbf{E} \times \mathbf{B}^*)]. \tag{89}$$

The ratio of the square of the angular momentum, M^2, to the square of the energy, U^2, for a pure (l, m) multipole [9, pp. 739–779] is

$$M^2/U^2 = m^2/\omega^2. \tag{90}$$

Experimentally, the photon can carry $\pm\hbar$ units of angular momentum. Thus, during excitation the spin, orbital, or total angular momentum of the orbitsphere can change by zero or $\pm\hbar$. The electron transition rules arise from conservation of angular momentum. The selection rules for multipole transitions between quantum states arise from conservation of total angular momentum and component angular momentum where the photon carries $\hbar$ of angular momentum. The radiation of a multipole of order (l, m) carries $m\hbar$ units of the z component of angular momentum per photon of energy $\hbar\omega$.

Consider the angular frequency of the electron orbitsphere. Given time harmonic motion and a radial delta function, the relationship between an allowed radius and the electron wavelength is given by Eq. (74). Using the de Broglie relationship for the electron momentum where the coordinates are spherical,

$$\lambda_n = h/p_n = h/m_e v_n \tag{91}$$

and the magnitude of the velocity for *every* point on the orbitsphere is

$$v_n = \hbar/m_e r_n. \tag{92}$$

The corresponding angular frequencies are related to r_n by

$$v_n = r_n\omega_n. \tag{93}$$

Thus,

$$\omega_n = \hbar/m_e r_n^2. \tag{94}$$

The sum of the $\mathbf{L}_i$, the magnitude of the angular momentum of each infinitesimal point of the orbitsphere of mass m_i, must be constant. The constant is $\hbar$.

$$\sum |\mathbf{L}_i| = \sum |\mathbf{r} \times m_i\mathbf{v}| = m_e r_n \frac{\hbar}{m_e r_n} = \hbar, \tag{95}$$

where the velocity is given by Eq. (92). The vector projections of the orbitsphere spin angular momentum relative to the Cartesian coordinates are given in the "Spin angular momentum of the Orbitsphere with $\ell = 0$" section of Mills [2]. The result is that the electron possesses a projection of the angular momentum onto an axis S of $\pm\sqrt{\frac{3}{4}}\hbar$ which precesses about the axis of an applied magnetic field at the Larmor frequency.

2.10. Photon absorption

In order that the excitation can occur, the correspondence principle holds such that the frequency of the photon that is absorbed is equal to the change in angular frequency of the electron. The energy of the photon which excites a mode in a stationary spherical resonator cavity from radius a_H to radius na_H is

$$E_{\text{photon}} = \frac{e^2}{4\pi\varepsilon_0 a_H}\left[1 - \frac{1}{n^2}\right] = h\nu = \hbar\omega, \tag{96}$$

where n = integer. After multiplying Eq. (96) by $a_H/a_H = 4\pi\varepsilon_0\hbar^2/e^2\mu_e a_H$, where a_H is the radius of the hydrogen atom,

ω_{photon} is

$$\omega_{photon} = \frac{\hbar}{m_e a_H^2}\left[1 - \frac{1}{n^2}\right]. \quad (97)$$

In the case of an electron orbitsphere, the resonator possesses kinetic energy before and after the excitation. The kinetic energy is always one-half of the potential energy because the centripetal force is an inverse squared central force. As a result, the energy and angular frequency to excite an electron orbitsphere is only one-half of the values above, Eqs. (96) and (97). From Eq. (94), the angular velocity of an electron orbitsphere of radius na_H is

$$\omega_n = \frac{\hbar}{m_e(na_H)^2}. \quad (98)$$

The change in angular velocity of the orbitsphere for an excitation from $n = 1$ to $n = n$ is

$$\Delta\omega = \frac{\hbar}{m_e(a_H)^2} - \frac{\hbar}{m_e(na_H)^2} = \frac{\hbar}{m_e(a_H)^2}\left[1 - \frac{1}{n^2}\right]. \quad (99)$$

The kinetic energy change of the transition is

$$\frac{1}{2}m_e(\Delta v)^2 = \frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 a_H}\left[1 - \frac{1}{n^2}\right] = \frac{1}{2}\hbar\omega. \quad (100)$$

The change in angular velocity of the electron orbitsphere, Eq. (99), is identical to the angular velocity of the photon necessary for the excitation, ω_{photon} (Eq. (97)). The energy of the photon necessary to excite the equivalent transition in an atomic hydrogen electron orbitsphere is one-half of the excitation energy of the stationary cavity because the change in kinetic energy of the electron orbitsphere supplies one-half of the necessary energy. The change in the angular frequency of the orbitsphere during a transition and the angular frequency of the photon corresponding to the superposition of the free space photon and the photon corresponding to the kinetic energy change of the orbitsphere during a transition are equivalent. The correspondence principle holds. It can be demonstrated that the resonance condition between these frequencies is to be satisfied in order to have a net change of the energy field [60].

3. Superfluid helium ion mobility results and discussion

Experiments to study the effect of light on ion mobility have been conducted by Northby and Sanders [61,62], Zipfel and Sanders [63,64], and Grimes and Adams [65,66]. For example, in the Northby and Sanders experiments [61,62], ions were introduced into the liquid from a radioactive source, and had to pass through two grids in order to reach the detector. The voltages on the grids were varied in time in a way such that normal negative ions could not reach the detector. It was found that when the liquid was illuminated, a small ion current reached the detector. Thus, they observed an increase in ion mobility under illumination, but recognized that the origin of the effect was unclear. It appears that the absorption of a photon by an electron bubble or orbitsphere in superfluid helium provides a natural explanation for the majority of the photo-conductivity results.

The photon absorption is determined by the correspondence principle. Thus, the radius of the electron following the absorption of a resonant photon is given by $n =$ (1/integer) times that of the original radius.

$$r = nr_1, \quad (101)$$

where $n = 1$/integer and r_1 is the radius of the electron in superfluid helium which has not absorbed a photon. This radius is determined by a force balance between the van der Waals pressure (force per unit area) of superfluid helium and the centrifugal force of the electron. The latter is given by

$$\mathbf{F}_{centrifugal} = \frac{m_e}{4\pi r_1^2}\frac{v_1^2}{r_1}, \quad (102)$$

where $m_e/4\pi r_1^2$ is the mass density of the orbitsphere and v_1 is given by Eq. (92). The radius r_1 can be determined from the photo-conductivity experiments of Zipfel and Sanders [64]. At zero pressure a photo-conductivity peak was observed at approximately 0.5 eV. From Eq. (97), the change in the frequency of the electron which matches frequency of the exciting photon is given by

$$\omega_{photon} = \frac{\hbar}{m_e r_1^2}\left[\frac{1}{n^2} - 1\right] \quad (103)$$

where $n = 1$/integer. The radius r_1 is given by

$$r_1 = \sqrt{\frac{\hbar}{m_e\omega_{photon}}\left(\frac{1}{n^2} - 1\right)}. \quad (104)$$

The relationship between energy and angular frequency of a photon is given by Planck's equation.

$$E = \hbar\omega_{photon}. \quad (105)$$

The angular frequency corresponding to a photon of 0.5 eV is

$$\omega_{photon} = \frac{8.0 \times 10^{-20} J}{\hbar} = 7.6 \times 10^{14} \text{ rad/s}. \quad (106)$$

In the case that 0.5 eV is the lowest energy transition for an electron in superfluid helium, the $n = 1 \rightarrow n = \frac{1}{2}$ transition corresponds to $n = \frac{1}{2}$ in Eq. (103). From Eqs. (103) and (106), the radius r_1 is

$$r_1 = \sqrt{\frac{\hbar}{m_e(7.6 \times 10^{14} \text{ rad/s})}\left(\frac{1}{(\frac{1}{2})^2} - 1\right)}$$

$$= 6.7 \times 10^{-10} \text{ m} = 6.7 \text{ Å} \quad (107)$$

where $n = \frac{1}{2}$. Comparing the case of the electron of a hydrogen atom to the case of an electron in helium, no initial central Coulomb field due to a proton is present, and the electron increases in kinetic energy upon photon absorption. Thus, the energy required to cause a transition in the latter case is twice that of the former. The photon stores energy

in the electric field of the resonator mode and increases the potential energy of the electron. The potential is the sum of the binding energy and the kinetic energy. The corresponding photon wavelength that will be absorbed by the electron is 2.5 μm.

The radius calculated in Eq. (107), is an approximation since the energy due to the pressure volume work and the surface energy change of the bubble were neglected. The former is given by

$$P\int dV = \frac{4}{3}\pi(r_1^3 - r_n^3)P \tag{108}$$

where P is the applied pressure, the integral is over the volume of the bubble, and r_1 and r_n are the initial and final radii of the electron bubble. The latter is given by

$$\alpha\int dA = 4\pi(r_1^2 - r_n^2)\alpha, \tag{109}$$

where α is the surface energy of helium per unit area, the integral is over the surface of the bubble, and r_1 and r_n are the initial and final radii of the electron bubble.

The contribution of these terms can be estimated by comparing the next experimental photo-conductivity peak at higher energy compared to the prediction given by Eqs. (103) and (105). Northby and Sanders [61,62] found that in the range of 0.7 to 3 eV the photo-induced current had a peak when the photon energy was 1.21 eV at zero pressure. Zipfel and Sanders [63,64] confirmed the peak at 1.21 eV. In experiments similar to those of Northby and Sanders [61,62], Zipfel and Sanders [63,64] made measurements of the photo-conductivity as a function of pressure up to 16 bar. The photo-conductivity peak detected by Northby and Sanders [61,62] was found to shift to higher photon energies as the pressure increased. This is expected since the radius of the normal electron decreases and the corresponding initial angular frequency increases with increasing pressure. Thus, the transition angular frequencies and energies increase (Eq. (103)).

The next higher energy transition for an electron in superfluid helium is $n = 1 \rightarrow n = \frac{1}{3}$. The transition energy corresponds to $n = \frac{1}{3}$ in Eqs. (103) and (105). The calculated energy neglecting the energy due to the pressure volume work and the surface energy change of the bubble is

$$E = \hbar\omega_{photon} = \frac{\hbar^2}{m_e r_1^2}\left[\frac{1}{(\frac{1}{3})^2} - 1\right]$$

$$= \frac{\hbar^2}{m_e(6.7\times 10^{-10}\ \mathrm{m})^2}\left[\frac{1}{(\frac{1}{3})^2} - 1\right] = 1.3\ \mathrm{eV}, \tag{110}$$

where r_1 is given by Eq. (107). Given the experimental uncertainty of the energy of the lowest energy transition, 1.21 eV, this result confirms that the contributions due to pressure volume work and the surface energy change of the bubble may be neglected.

In the experiments of Northby and Sanders [61,62], Zipfel and Sanders [63,64], and Grimes and Adams [65,65], it was noted that the photo-conductivity effect was absent above a critical temperature. This temperature was approximately 1.7 K at zero pressure, and decreased to 1.2 K at 20 bar. Roton scattering dominates over phonon scattering at 1.7 K and below [59]. The photo-conductivity signal disappears because of phonon excitation of the bubble motion which causes the excited electron state to decay. As the pressure is increased, the roton energy gap goes down, and so the phonon scattering increases. Thus, it is to be expected that the critical temperature decreases with increasing pressure.

Each stable excited state electron bubble which has a radius of r_1/integer may migrate in an applied electric field. The bubble may be scattered by rotons, phonons, and He^3 impurities. At temperatures less that 1.7 K, roton scattering dominates [59]. An equation for the electron bubble mobility is derived by Baym et al. [67] in terms of the roton-bubble momentum transfer cross section by calculating the rate of roton-bubble momentum transfer using a statistical mechanical approach. In the case of an elementary excitation $\vec{k}$ scattered by the bubble with a differential cross section $\sigma(k,\theta)$ and obeying $|\vec{k}'| \cong |\vec{k}|$, their result may be written

$$\frac{e}{\mu} = -\frac{\hbar^2}{6\pi^2}\int_0^\infty k^4\frac{\delta n}{\delta\varepsilon}v_g(k)\sigma_T(k)\,dk \tag{111}$$

where μ is the bubble mobility, n is the distribution function of the excitation, $v_g(k)$ is the group velocity of the excitation, and $\sigma_T(k)$ is the momentum-transfer cross section defined by

$$\sigma_T(k) = \int(1-\cos\theta)\sigma(k,\theta)\,d\Omega. \tag{112}$$

Schwarz and Stark [59] made the reasonable assumption that $\sigma_T(k)$ is a weak function of $k - k_0$. Because of the strong minimum at $k_0 = 1.91\ \text{Å}^{-1}$ in the roton energy spectrum, Eq. (111) then gives to a good approximation

$$\mu = \frac{3\pi^2 e}{\hbar k_0^4\sigma_T(k_0)}\exp(\Delta/k_B T)$$

$$= \frac{3.38\times 10^{-25}\ \mathrm{m^4\ V^{-1}\ s^{-1}}}{\sigma_T(k_0)}\exp(8.65\ \mathrm{K/T}) \tag{113}$$

where $\Delta/k_B = 8.65 \pm 0.04$ K is the roton energy gap derived from neutron scattering [68]. Schwarz and Stark [59] propose that the roton de Broglie wavelength corresponding to $k_0 = 1.91\ \text{Å}^{-1}$ is $\lambda_0 = 3.3$ which is small compared with $\sqrt{\sigma_T(k_0)/\pi}$; thus, the collision cross section may be nearly geometrical. Although the roton carries a great deal of energy and momentum, its effective mass is much less than that of the ion. Assume that the scattering is elastic, then $|\vec{k}'| \cong |\vec{k}|$ is satisfied. They conclude a hard-sphere cross section given by

$$\sigma_T(k_0) \cong \pi(a_+ + a_r)^2 \tag{114}$$

where a_+ is the radius of the ion and a_r is the effective collision radius of the roton. Using experimental values for a_+ and $\sigma_T(k)$, they find that

$$a_r = 3.7 \pm 0.2\ \text{Å}. \tag{115}$$

They surmise from this that the roton is localized within a region of radius $\approx 3.7-4.0$ Å, and that it interacts strongly with any disturbance which penetrates this region. They point out that $\approx 3.7-4.0$ Å is only slightly larger than the nearest neighbor distance in liquid helium [69] and that a roton may thus be pictured as a highly correlated motion of an energetic He^4 atom and its nearest neighbors only.

The geometric cross section of the normal electron bubble σ_e is given as

$$\sigma_e = \pi r_1^2 \tag{116}$$

where r_1 is the radius of the unexcited electron bubble given by Eq. (107). From Eq. (107) and Eqs. (113)–(116), the mobility of the normal electron bubble is given by

$$\mu = \frac{3\pi^2 e}{\hbar k_0^4 \pi (a_r + r_1)^2} \exp(\Delta/k_B T)$$

$$= \frac{3.38 \times 10^{-25} \mathrm{m^4/V\,s}}{\pi(3.7 \times 10^{-10}\mathrm{m} + 6.7 \times 10^{-10}\,\mathrm{m})^2} \exp(8.65\,\mathrm{K/T}) \tag{117}$$

At 1 K, Eq. (117) gives $\mu = 5.7\ \mathrm{cm^2/V\,s}$ for the mobility of the normal electron bubble ($n = 1$) which is in reasonable agreement with the experimental value of $5\ \mathrm{cm^2/V\,s}$ from Fig. 6.

The normal electron bubble has a uniform constant spherical charge density. This charge density may be modulated by a time and spherically harmonic function as given by Eq. (53).[1] In the case of excited state electron bubbles, the contribution to the roton scattering cross section given by Eq. (112) is larger than the geometric cross section given in Eq. (116) where the radius is given by Eq. (101). In this case, $\sigma_T(k)$ given by Eq. (112) follows the derivation of Baym et al. [67] where the spherical harmonic angular function causes a gain in the scattering cross section that may be modeled after that of a Hertzian dipole antenna. The radiation power pattern of a Hertzian dipole is given by Kong [70]. The radiation power pattern is

$$\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}[\mathbf{E} \times \mathbf{H}] = \hat{r}\frac{\eta}{2}\left(\frac{k|I|\Delta z}{4\pi r}\right)\sin^2\theta \tag{118}$$

where I is the current, Δz is the length of the dipole, and η is the impedance of free space. The antenna directive gain $D(\theta, \varphi)$ is defined as the radiation of the Poynting power density $\langle S_r \rangle$ over the power P, divided by the area of the sphere:

$$D(\theta, \phi) = \frac{\langle S_r \rangle}{P/4\pi r^2} = \frac{3}{2}\sin^2\theta. \tag{119}$$

The plot of $D(\theta, \varphi)$ given by Eq. (119) is known as the gain pattern. The directivity of an antenna is defined as the value of the gain in the direction of its maximum value. For the Hertzian dipole the maximum of 1.5 occurs at $\theta = \pi/2$. Thus, the directivity of a Hertzian dipole is 1.5.

The spherical harmonic angular functions are

$$Y_{lm}(\theta, \phi) = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P_l^m(\cos\theta) e^{im\phi}$$

$$= N_{l,m_l} P_l^m(\cos\theta) e^{im\phi} \tag{120}$$

where N_{l,m_l} is the normalization constant given by

$$N_{l,m_l} = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}}. \tag{121}$$

In the case of excited states, $\sigma(k, \theta)$ of Eq. (112) is

$$\sigma(k, \theta) = k^{-2}\left|\frac{\int P_0^0(\cos\theta)e^{i0\phi}\,d\Omega}{\int P_l^m(\cos\theta)e^{im\phi}\,d\Omega}\right|^2 = k^{-2}\left(\frac{N_{l,m_l}}{N_{0,0}}\right)^2. \tag{122}$$

For excited states, the geometric cross section of the electron bubble σ_e is then given as

$$\sigma_e = \pi n r_{n,l,m_l}^2 \tag{123}$$

where

$$r_{n,l,m_l} = \frac{N_{l,m_l}}{N_{0,0}} n r_1. \tag{124}$$

r_1 is the radius of the unexcited electron bubble given by Eq. (107) and n = 1/integer. The angular parameters $N_{l,m_l}/N_{0,0}$ are given with the first few spherical harmonics in Table 1. In this case, $\sigma_T(k)$ is given by Eq. (112) where r_1 is replaced by r_{n,l,m_l} (Eq. (124)). The roton scattering cross section given by the hard-sphere cross section is then

$$\sigma_T(k_0) \cong \pi(r_{n,l,m_l} + a_r)^2, \tag{125}$$

where a_r is the effective collision radius of the roton given by Eq. (115). From Eqs. (117), (124) and (125), the mobilities

[1] It is interesting to consider that the solutions for the full angular part of the Schrödinger equation (Eq. (4)), $Y_{lm}(\theta, \phi)$, are also the spherical harmonics. McQuarrie [8, pp. 206–221] shows that the Schrödinger equation for the rigid rotor is

$$\hat{H} Y_l^{m_l}(\theta, \phi) = \frac{\hbar^2 l(l+1)}{2I} Y_l^{m_l}(\theta, \phi) \tag{127}$$

and that $\hat{H}$ and $\hat{L}^2$ differ only by the factor $2I$ for a rigid rotor. So, Eq. (127) is equivalent to

$$\hat{L}^2 Y_l^{m_l}(\theta, \phi) = \hbar^2 l(l+1) Y_l^{m_l}(\theta, \phi). \tag{128}$$

Thus, we see that the spherical harmonics are also eigenfunctions of $\hat{L}^2$ and that the square of the angular momentum can have values given by

$$L^2 = \hbar^2 l(l+1) \quad l = 0, 1, 2, 3, \ldots . \tag{129}$$

The flaw with this result with regard to the hydrogen atom is given in the "Schrödinger Theory of the Hydrogen Atom" section. In the case of the electron in helium, the moment of inertia in Eq. (127) decreases with increasing energy states as the corresponding l quantum number increases. The Schrödinger equation is flawed based on the prediction of infinite rotational energy for an electron in superfluid helium.

Table 1
The first few spherical harmonics and $N_{\ell,m_\ell}/N_{0,0}$ of Eq. (121) as a function of ℓ, and m_ℓ

Spherical harmonics $Y^{\ell}_{m_\ell}$	ℓ	m_ℓ	$N_{\ell,m_\ell}/N_{0,0}$
$Y^0_0 = \frac{1}{(4\pi)^{1/2}}$	0	0	1
$Y^0_1 = (\frac{3}{4\pi})^{1/2}\cos\theta$	1	0	$\sqrt{3}$
$Y^1_1 = (\frac{3}{8\pi})^{1/2}\sin\theta e^{i\phi}$	1	1	$\sqrt{\frac{3}{2}}$
$Y^{-1}_1 = (\frac{3}{8\pi})^{1/2}\sin\theta e^{-i\phi}$	1	−1	$\sqrt{\frac{3}{2}}$
$Y^0_2 = (\frac{5}{16\pi})^{1/2}(3\cos^2\theta - 1)$	2	0	$\sqrt{\frac{5}{4}}$
$Y^1_2 = (\frac{15}{8\pi})^{1/2}(\sin\theta\cos\theta e^{i\phi})$	2	1	$\sqrt{\frac{15}{2}}$
$Y^{-1}_2 = (\frac{15}{8\pi})^{1/2}(\sin\theta\cos\theta e^{-i\phi})$	2	−1	$\sqrt{\frac{15}{2}}$
$Y^2_2 = (\frac{15}{32\pi})^{1/2}\sin^2\theta e^{2i\phi}$	2	2	$\sqrt{\frac{15}{8}}$
$Y^{-2}_2 = (\frac{15}{32\pi})^{1/2}\sin^2\theta e^{-2i\phi}$	2	−2	$\sqrt{\frac{15}{8}}$

of electron bubbles are given by

$$\mu = \frac{3\pi^2 e}{\hbar k_0^4 \pi (a_r + r_{n,\ell,m_\ell})^2}\exp(\Delta/k_B T)$$
$$= \frac{3.38\times 10^{-25}\,\mathrm{m^4/V\,s}}{\pi(3.7\times 10^{-10}\mathrm{m} + n\frac{N_{\ell,m_\ell}}{N_{0,0}}6.7\times 10^{-10}\mathrm{m})^2}\exp(8.65\,\mathrm{K/T}) \quad (126)$$

where n = 1/integer. The mobility of an excited state electron bubble having a fractional principal quantum number (n = 1/integer) relative to the normal electron bubble as a function of quantum numbers n, ℓ, and m_ℓ is given in Table 2. A plot of Eq. (126) normalized to the mobility of the normal bubble as a function of p corresponding to fractional principal quantum number n = 1/integer = $1/p$ for given ℓ, and m_ℓ quantum numbers appears in Fig. 5.

Using time of flight, Doake and Gribbon [71] detected negatively charged ions that had a mobility substantially higher than the normal electron bubble negative ion. This ion, which has become known as the "fast ion", was next seen in another time-of-flight experiment by Ihas and Sanders in 1971 [72]. They showed that the fast ion could be produced by an α or β source, or by an electrical discharge in the helium vapor above the liquid. In addition, they reported the existence of two additional negative carriers, referred to as "exotic ions", that had mobilities larger than the mobility of the normal negative ion, but less than the mobility of the fast ion. These exotic ions were detected only when there was an electrical discharge above the liquid surface. In a paper the following year [73], Ihas and Sanders (IS) reported on further experiments in which at least 13 carriers with different mobilities were detected. The experimental details are described in the thesis of Ihas [74]. Eden and McClintock (EM) [75,76] also detected as many as 13 ions with different mobilities. Both IS and EM put forward a number of proposals to explain the exotic ions, but all of these proposals were shown to be unsatisfactory by Maris [1]. It is significant that the exotic ions appear only when an electrical discharge takes place close to the free surface of the liquid. Under these conditions, the electrons that enter the liquid and form bubbles may absorb light emitted from the discharge. Thus, it is natural to consider the possibility that the exotic ions are electron bubbles in fractional energy states.

The mobility of several electron bubbles in superfluid helium plotted versus the inverse of the temperature is shown in Fig. 6. The temperature dependence of the mobility predicted by Eq. (126) is in good agreement with the data of Ihas [74] and the plots of Maris [1]. The ion assignments given in Fig. 6 are based on their mobilities relative to the normal ion as given in Table 3.

Table 2
The mobility of an excited state electron bubble having a fraction principal quantum number (n = 1/integer) relative to the normal electron bubble as a function of quantum numbers n, ℓ, and m_ℓ given by Eq. (126). The peaks that appear in Fig. 7 and Table 3 are indicated

n	$\ell = 0$	$\ell = 1\ m_\ell = 0$	$\ell = 1\ m_\ell = \pm 1$	$\ell = 2\ m_\ell = 0$	$\ell = 2\ m_\ell = \pm 1$	$\ell = 2\ m_\ell = \pm 2$
$\frac{1}{2}$	2.21 Peak # 8	1.22 Peak # 3	1.81 Peak # 5			
$\frac{1}{3}$	3.12 Peak # 10	1.92 Peak # 6	2.66	2.86	1.14 Peak # 2	2.41
$\frac{1}{4}$	3.81 Peak # 11	2.52 Peak # 9	3.33	3.54	1.60 Peak # 4	3.06
$\frac{1}{5}$	4.33 Peak # 12	3.03	3.86	4.07	2.03 Peak # 7	3.59
$\frac{1}{6}$	4.74 Peak # 13	3.47	4.28	4.49	2.41	4.02
$\frac{1}{7}$	5.07 Peak # 14	3.83	4.63	4.83	2.75	4.38
$\frac{1}{8}$	5.34 Peak # 15	4.15	4.93	5.12	3.06	4.68
$\frac{1}{9}$	5.57 Peak # 15	4.42	5.17	5.35	3.34	4.94
$\frac{1}{10}$	5.76 Peak # 15	4.66	5.38	5.56	3.59	5.16
$\frac{1}{11}$	5.92 Peak # 15	4.87	5.56	5.73	3.82	5.35
$\frac{1}{12}$	6.07 Peak # 15	5.05	5.72	5.88	4.02	5.52
$\frac{1}{100}$	7.75 Peak # 15	7.55	7.69	7.72	7.29	7.65

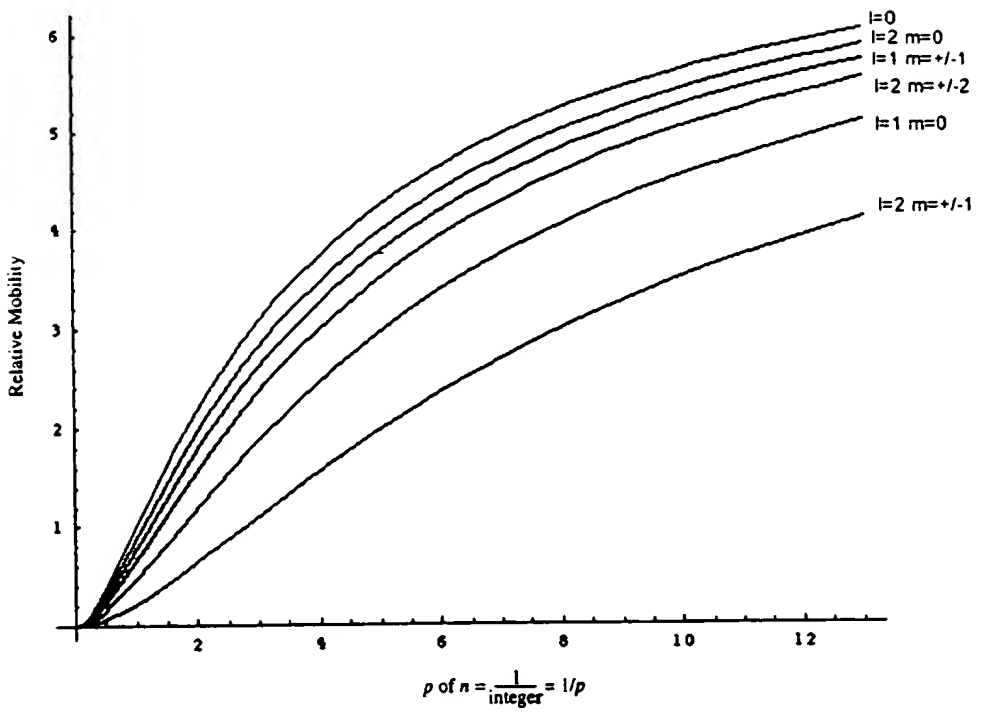

Fig. 5. The mobility (Eq. (126)) of an excited state electron bubble having a fraction principal quantum number (n = 1/integer = $1/p$) relative to the normal electron bubble as a function of p for given ℓ, and m_ℓ quantum numbers.

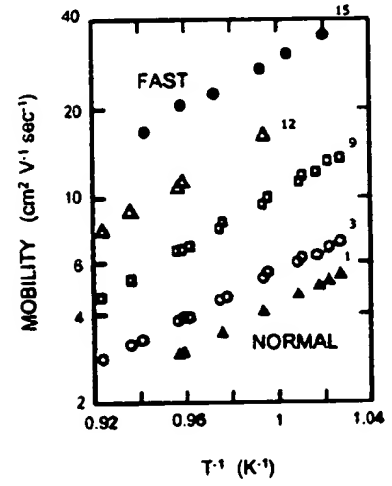

Fig. 6. The mobility from the data of Ihas [1,74] of electron bubbles in superfluid helium plotted versus the inverse of the temperature. Solid triangles are for the normal electron bubble, open squares, circles, triangles and solid circles are for the four ions of the same peak assigned in Fig. 7 and Table 3.

Following a pulse discharge with an electric field applied to superfluid helium, Ihas [74] recorded ion peaks using time of flight. Fifteen ion peaks recorded by Ihas and Sanders are identified in Fig. 7. The mobilities relative to the normal electron bubble (n = 1) are given in Table 3. The assignments of the mobilities of excited state electron bubbles having fraction principal quantum number (n = 1/integer) relative to the normal electron bubble as a function of quantum numbers n, ℓ, and m_ℓ is also given in Table 3 based on the theoretical valves given in Table 2. The agreement between theory and experiment is excellent.

Peaks 14–15 of Fig. 7 and Table 3 represent a band with a cutoff at a migration velocity of about 7.5 times the velocity of the normal ion as n = 1/integer approaches zero (n = 1/100 was used to calculate this limiting case). The electron radius is predicted to decrease such that the effective collision radius of the roton determines the maximum mobility as given by Eq. (126). The theoretically predicted maximum of electron bubble mobility of about seven times that of the normal ion is confirmed by the Ihas data [74] where the band comprising peaks 14–15 correspond to $n \leq \frac{1}{7}$. Furthermore, Eden and McClintock [75] and Doake and Gribbon [71] measured the drift velocity as a function of applied electric field. The fast ion showed a slope of the drift velocity versus applied electric field of about seven times

Table 3
The migration times and experimental mobilities of the 15 ion peaks shown in Fig. 7 relative to the normal ion with their assignments to excited state electron bubbles with quantum numbers n, ℓ, and m_ℓ and theoretical mobilities given in Table 2

Peak #	Migration time (arbitrary units)	Mobility relative to peak # 1	Theoretical mobility relative to Peak # 1	Assignment n, ℓ, and m_ℓ
1	9.8	1.00	1	$n = 1\ \ell = 0\ m_\ell = 0$
2	8.2	1.20	1.14	$n = \frac{1}{3}\ \ell = 2\ m_\ell = \pm 1$
3	7.6	1.29	1.22	$n = \frac{1}{2}\ \ell = 1\ m_\ell = 0$
4	6.2	1.58	1.6	$n = \frac{1}{4}\ \ell = 2\ m_\ell = \pm 1$
5	5.4	1.81	1.81	$n = \frac{1}{2}\ \ell = 1\ m_\ell = \pm 1$
6	5	1.96	1.92	$n = \frac{1}{3}\ \ell = 1\ m_\ell = 0$
7	4.85	2.02	2.03	$n = \frac{1}{3}\ \ell = 2\ m_\ell = \pm 1$
8	4.35	2.25	2.21	$n = \frac{1}{2}\ \ell = 0\ m_\ell = 0$
9	3.9	2.51	2.52	$n = \frac{1}{4}\ \ell = 1\ m_\ell = 0$
10	3.3	2.97	3.12	$n = \frac{1}{3}\ \ell = 0\ m_\ell = 0$
11	2.8	3.50	3.81	$n = \frac{1}{4}\ \ell = 0\ m_\ell = 0$
12	2.1	4.67	4.33	$n = \frac{1}{5}\ \ell = 0\ m_\ell = 0$
13	2	4.90	4.74	$n = \frac{1}{6}\ \ell = 0\ m_\ell = 0$
14	1.8	5.44	5.07	$n = \frac{1}{7}\ \ell = 0\ m_\ell = 0$
15	1.3	7.54	7.75	$n = \frac{1}{100}\ \ell = 0\ m_\ell = 0$

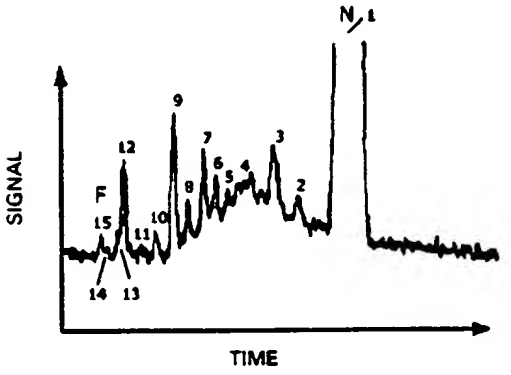

Fig. 7. Data trace from Ihas [74] showing the detected ion signal as a function of time. N and F denote the normal and fast ion peaks. The peaks labeled 1–15 are assigned in Table 3. For a description of experimental condition see Ihas [74].

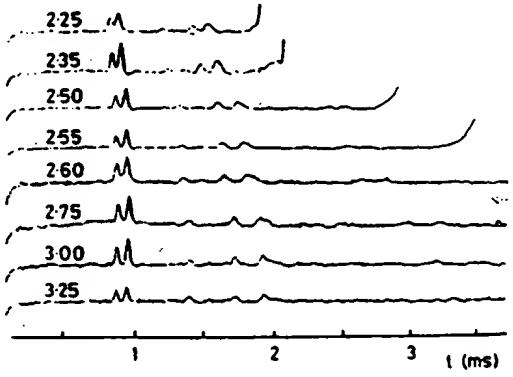

Fig. 8. Data traces from Eden and McClintock [75] of the current at the collector of the velocity spectrometer (arbitrary units) as a function of the elapsed time t after a pulse was applied to release exotic ions. Signals are shown for a range of strong electric fields as indicated in units of 10^5 V/m by the number above each trace. The steep rise on the right-hand sides of the signals indicates the arrival of the normal ion current. For a description of experimental condition see Eden and McClintock [75].

that of the normal ion. Thus, these results agree with the data of Ihas and with theoretical predictions.

The small deviation of the data from the theoretical in Table 3 may be due to differences in ion production rates and mechanisms based on the spectrum of the arc. Transitions between states may also be a peak broadening factor wherein a peak undergoes a transition to a faster or slower state during migration. This may provide an explanation for the large peak width of peak #4 of Fig. 7 as well as the broad continuum background in this region. Scattering other than roton scattering may also be involved, and these mechanisms such as phonon scattering and inter-bubble "impurity" scattering would effect larger ions more than smaller ions due to their larger geometric cross section. A preferred method to determine the migration times of each electron bubble ($n = 1$/integer) is to cause the formation of each specific state with resonant radiation (Eq. (103) and Eq. (105))

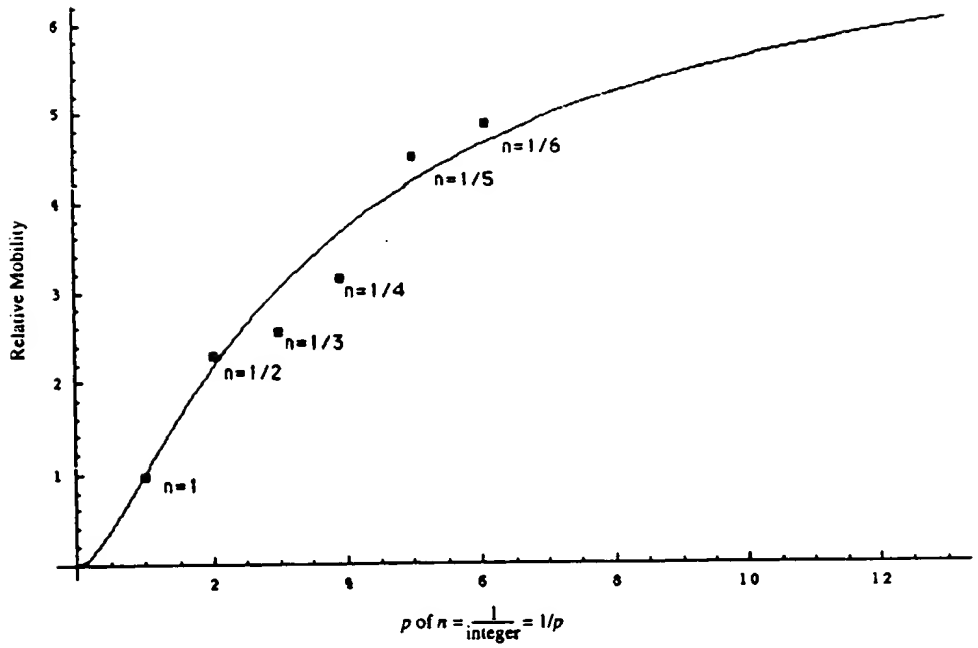

Fig. 9. The limiting electron bubble velocities shown in the data traces of Fig. 8 relative to the normal electron bubble as a function of p corresponding to principal quantum number $n = 1/\text{integer} = 1/p$ where $\ell = 0$, and $m_\ell = 0$.

and to measure the migration time of each ion separately relative to the $n = 1$ bubble.

Using a time-of-flight method, Doake and Gribbon [71] discovered that a fast ion can exist in superfluid helium which unlike the normal ion may be accelerated to the Landau critical velocity v_L for roton creation without undergoing metamorphosis to a charged vortex ring, even under the saturated vapor pressure. As a consequence, Eden and McClintock [75] studied the behavior of the exotic ions in strong electric field and reported evidence suggesting that intermediate mobility negative ions can nucleate quantized vortex rings in superfluid helium when subject to strong electric fields. Eden and McClintock observed that the drift velocity of intermediate ions may not be linear with electric field and that this effect varies with the particular intermediate ion. Eden and McClintock [75] further observed a decrease in drift velocities of intermediate ions with inter-ion variation for increasing sufficiently strong electric fields. They attributed this to the formation of negatively charged vortex rings. However, for an appropriate electric field, the drift velocity approaches a maximum independent of the field, and formation of charged vortex rings does not explain the field independence [75]. The limiting velocities shown in the data traces of Fig. 8 may be a function of the size of the ion for all intermediate ions. In this case, the limiting velocity data of Eden and McClintock [75] are plotted in Fig. 9 with the mobility of the normal ion as a function of p corresponding to principal quantum number $n = 1/\text{integer} = 1/p$. The agreement between the experimental data and theoretical mobilities is excellent. The existence of multiple peaks under the fast peak such as peaks #14 and #15 of Fig. 7 is also supported by the data of Eden and McClintock [75] because the peak of highest mobility split into the two peaks shown in Fig. 8 at higher fields.

4. Conclusion

Recently, the behavior of free electrons in superfluid helium has again forced the issue of the meaning of the wave function. Electrons form bubbles in superfluid helium which reveal that the electron is real and that a physical interpretation of the wave function is necessary. Furthermore, when irradiated with low energy light, the electrons carry increased current at different rates as if they exist with at least 15 different sizes. Peaks are observed in the photoconductivity absorption spectrum at 0.5 and 1.21 eV. A theory of classical quantum mechanics derived from first principles by Mills [2] gives closed form physical solutions for the electron in atoms, the free electron, and the free electron in superfluid helium. The predicted photo-conductivity absorption spectrum and the mobilities of the 15 identified ions match the experimental results. The data support the existence of fractional principal quantum energy states of free electrons in superfluid helium predicted by Mills classical theory. The results have implications that the concept

of probability waves of quantum mechanics must be abandoned and atomic theory must be based in reality.

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Special thanks to Professor T.N. Veziroglu for bringing Maris's paper to my attention.

Appendix A.

A.1. Quantum electrodynamics is purely mathematical and has no basis in reality

Quantum mechanics failed to predict the results of the Stern–Gerlach experiment which indicated the need for an additional quantum number. Quantum electrodynamics was proposed by Dirac in 1926 to provide a generalization of quantum mechanics for high energies in conformity with the theory of special relativity and to provide a consistent treatment of the interaction of matter with radiation. From Weisskopf [18], "Dirac's quantum electrodynamics gave a more consistent derivation of the results of the correspondence principle, but it also brought about a number of new and serious difficulties". Quantum electrodynamics: (1) does not explain nonradiation of bound electrons; (2) contains an internal inconsistency with special relativity regarding the classical electron radius—the electron mass corresponding to its electric energy is infinite; (3) it admits solutions of negative rest mass and negative kinetic energy; (4) the interaction of the electron with the predicted zero-point field fluctuations leads to infinite kinetic energy and infinite electron mass; (5) Dirac used the unacceptable states of negative mass for the description of the vacuum; yet, infinities still arise. In 1947, contrary to Drac's predictions Lamb discovered a 1000 MHz shift between the $^2S_{1/2}$ state and the $^2P_{1/2}$ state of the hydrogen atom [113]. This so called Lamb shift marked the beginning of modern quantum electrodynamics. In the words of Dirac [77], "No progress was made for 20 years. Then a development came initiated by Lamb's discovery and explanation of the Lamb Shift, which fundamentally changed the character of theoretical physics. It involved setting up rules for discarding ... infinities ...". Renormalization is presently believed to be required of any fundamental theory of physics [78].

However, dissatisfaction with renormalization has been expressed at various times by many physicists including Dirac [79] who felt that, "This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it turns out to be small—not neglecting it just because it is infinitely great and you do not want it!"

Furthermore, Oskar Klein pointed out a glaring paradox implied by the Dirac equation which was never resolved [80]. "Electrons may penetrate an electrostatic barrier even when their kinetic energy, $E - mc^2$ is lower than the barrier. Since in Klein's example the barrier was infinitely broad this could not be associated with wave mechanical tunnel effect. It is truly a paradox: Electrons too slow to surpass the potential, may still only be partially reflected. ... Even for an infinitely high barrier, i.e. $r_2 = 1$ and energies ≈ 1 MeV (the reflection coefficient) R is less than 75%! From (2) and (3) it appears that as soon as the barrier is sufficiently high: $V > 2mc^2$, electrons may transgress the repulsive wall—seemingly defying conservation of energy. ... Nor is it possible by way of the positive energy spectrum of the free electron to achieve complete Einstein causality".

The Rutherford experiment demonstrated that even atoms are comprised of essentially empty space [81]. Zero-point field fluctuations, virtual particles, and states of negative energy and mass invoked to describe the vacuum are nonsensical and have no basis in reality since they have never been observed experimentally and would correspond to an essentially infinite cosmological constant throughout the entire universe including regions of no mass. As given by Waldrop [82], "What makes this problem into something more than metaphysics is that the cosmological constant is observationally zero to a very high degree of accuracy. And yet, ordinary quantum field theory predicts that it ought to be enormous, about 120 orders of magnitude larger than the best observational limit. Moreover, this prediction is almost inescapable because it is a straightforward application of the uncertainty principle, which in this case states that every quantum field contains a certain, irreducible amount of energy even in empty space. Electrons, photons, quarks—the quantum field of every particle contributes. And that energy is exactly equivalent to the kind of pressure described by the cosmological constant. The cosmological constant has accordingly been an embarrassment and a frustration to every physicist who has ever grappled with it".

The spin of the electron and the Lamb shift are calculated from first principles in closed form by Mills [2]. The spin angular momentum results from the motion of negatively charged mass moving systematically, and the equation for angular momentum, $\mathbf{r} \times \mathbf{p}$, can be applied directly to the wave function (a current density function) that describes the electron. The Lamb shift results from conservation of linear momentum of the photon.

A.2. The postulate of quantum measurement is experimentally disproved

Modern quantum mechanics has encountered several obstacles that have proved insurmountable as pointed out previously in the "General considerations" section and the "Classical electron radius" section of Mills [2]. And, quantum mechanics leads to certain philosophical interpretations [17, pp. 94–104] which are not sensible. Some conjure up multitudes of universes including "mind" universes; others require belief in a logic that allows two contradictory statements to be true. The question addressed is whether the

universe is determined or influenced by the possibility of our being conscious of it.

The meaning of quantum mechanics is debated, but the Copenhagen interpretation is predominant. It asserts that "what we observe is all we can know; any speculation about what a photon, an atom or even a SQUID (Superconducting Quantum Interference Device) really is or what it is doing when we are not looking is just that speculation" [17, pp. 94–104]. According to this interpretation every observable exists in a state of superposition of possible states, and observation or the potential for knowledge causes the wave function corresponding to the possibilities to collapse into a definite. As shown by Platt [83] in the case of the Stern–Gerlach experiment, "the postulate of quantum measurement [which] asserts that the process of measuring an observable forces the state vector of the system into an eigenvector of that observable, and the value measured will be the eigenvalue of that eigenvector".

According to the Zeno no-go theorem which is a consequence of the postulate of quantum measurement, observation of an atom collapses its state into a definite; thus, transitions cannot occur under continuous observation. Recently, it has become possible to test this postulate via an experiment involving transitions of a single atom, and the results are inconsistent with the predictions. Quoting from the caption of Fig. 10 of the article, by Dehmelt [84],

> "Shelving" the Ba^+ optical electron in the metastable D level. Illuminating the ion with a laser tuned close to its resonance line produces strong resonance fluorescence and an easily detectable photon count of 1600 photons/s. When later an auxiliary weak Ba^+ spectral lamp is turned on, the ion is randomly transported into the metastable $D_{5/2}$ level for 30-s lifetime and becomes invisible. After dwelling in this shelving level for 30 s on average, it drops down to the S ground state spontaneously and becomes visible again. This cycle repeats randomly. According to the Zeno no-go theorem, no quantum jumps should occur under continuous observation.

In Scientific American [17, pp. 94–104] at "Superimposed philosophers"—"Pritchard says that physicists may one day be able to pass biologically significant molecules such as proteins or nucleic acids through an interferometer. In principle, one could even observe wave like behavior in a whole organism, such as an amoeba. There are some obstacles, though: the amoeba would have to travel very slowly, so slowly, in fact that it would take some three years to get through the interferometer, according to Pritchard. The experiment would also have to be conducted in an environment completely free of gravitation or other influences—that is, in outer space.

Getting a slightly larger or more intelligent organism, for instance, a philosopher, to take two paths through a two-slit apparatus would be even trickier. "It would take longer than the age of the universe," Pritchard says.".

This article is a good example of how far fetched QM has become (e.g. *parallel mind universes*).

In addition to the interpretation that photons, electrons, neutrons, and even human beings [17, pp. 94–104] have no definite form until they are measured, a more disturbing interpretation of quantum mechanics is that a measurement of a quantum entity can instantaneously influence another light years away. Einstein argued that a probabilistic versus deterministic nature of atomic particles leads to disagreement with special relativity. In fact, the nonlocality result of the Copenhagen interpretation violates causality. As a consequence of the indefinite nature of the universe according to quantum mechanics and the implied uncertainty principle, Einstein, Podolsky, and Rosen (EPR) in a classic paper [85] presented a paradox which led them to infer that quantum mechanics is not a complete theory. See the section entitled, "Heisenberg uncertainty principle predicts nonlocality, noncausality, spooky actions at a distance, and perpetual motion which can be shown to be experimentally incorrect".

Louis de Broglie, one of the founders of quantum mechanics, also condemned the probability wave approach. "But the causal link between two phenomenon implies the existence of a trajectory, and to deny this existence is to renounce causality and to deprive oneself of any understanding" [86].

A.3. Quantum mechanics based on the Schrödinger equation (SE) is an incomplete theory since it does not explain gravity or particle masses

Quantum mechanics cannot explain the existence of particles with precise masses and gives no basis of gravity. If fact, a straight forward application of the uncertainty principle predicts that particles of precise mass/energy cannot exist. These shortcomings are compounded by the prediction of zero-point field fluctuations, virtual particles, and states of negative energy and mass invoked to describe the vacuum. These consequences of the uncertainty principle are nonsensical and have no basis in reality since they have never been observed experimentally and would correspond to an essentially infinite cosmological constant throughout the entire universe including regions of no mass [87].

Mills [2] derives a theory from first principles that gives closed form solutions based on general relativity and Maxwell's equations for particle masses. It gives gravitation from the atom to the cosmos.

For any kind of wave advancing with limiting velocity and capable of transmitting signals, the equation of front propagation is the same as the equation for the front of a light wave. By applying this condition to electromagnetic and gravitational fields at particle production, the Schwarzschild metric (SM) is derived from the classical wave equation which modifies general relativity to include conservation of spacetime in addition to momentum and matter/energy. The result gives a natural relationship between Maxwell's equations, special relativity, and general relativity. It gives gravitation from the atom to the cosmos. The universe is time

harmonically oscillatory in matter energy and spacetime expansion and contraction with a minimum radius that is the gravitational radius. In closed form equations with fundamental constants only, CQM gives the deflection of light by stars, the precession of the perihelion of Mercury, the particle masses, the Hubble constant, the age of the universe, the observed acceleration of the expansion, the power of the universe, the power spectrum of the universe, the microwave background temperature, the uniformity of the microwave background radiation, the microkelvin spatial variation of the microwave background radiation, the observed violation of the GZK cutoff, the mass density, the large-scale structure of the universe, and the identity of dark matter which matches the criteria for the structure of galaxies. In a special case wherein the gravitational potential energy density of a blackhole equals that of the Plank mass, matter converts to energy and spacetime expands with the release of a gamma ray burst. The singularity in the SM is eliminated.

A.4. The wave function solutions of quantum mechanics as probability waves are inconsistent with probability theory

Wavefunction solutions of the Schrödinger equation are interpreted as probability density functions. This interpretation is fatally flawed since the use of "probability" in this instance does not conform to the mathematical rules and principles of probability theory.

Schrödinger sought a physical interpretation of his equation. He interpreted $e\Psi^*(x)\Psi(x)$ as the charge-density or the amount of charge between x and $x + dx$ (Ψ^* is the complex conjugate of Ψ). Presumably, then, he pictured the electron to be spread over large regions of space. Three years after Schrödinger's interpretation, Max Born, who was working with scattering theory, found that this interpretation led to logical difficulties, and he replaced the Schrödinger interpretation with the probability of finding the electron between r, θ, ϕ and $r + dr, \theta + d\theta, \phi + d\phi$ as

$$\int \Psi(r,\theta,\phi)\Psi^*(r,\theta,\phi)dr\,d\theta\,d\phi. \tag{A.1}$$

Born's interpretation is generally accepted. The electron *is viewed as a discrete particle* that moves here and there (from $r = 0$ to $r = \infty$), and $\Psi\Psi^*$ gives the time average of this *motion.*

Born's interpretation is generally accepted. Nonetheless, interpretation of the wave function is a never-ending source of confusion and conflict. Many scientists have solved this problem by conveniently adopting the Schrödinger interpretation for some problems and the Born interpretation for others. This duality allows the electron to be everywhere at one time—yet have no volume. Alternatively, the electron can be viewed as a discrete particle that moves here and there (from $r = 0$ to $r = \infty$), and $\Psi\Psi^*$ gives the time average of this motion.

There is profound internal inconsistency in using probability theory as the foundation of quantum theory. In the tradition of Einstein, a Gedanken experiment is proposed. At a given time, there exists an exact average of the amount of money in the wallets of the members of a given group such as the visitors to BLP's web site. So, I postulate some statistical test and sample 0 to all of the visitors. If I sample all of the visitors, I would know the average exactly. If I sample less than all, I could apply statistics. Using the sampling data with the assumption of a statistical distribution (e.g. *normal distribution*), I could perform a two-sided null test and determine the average within a confidence limit based on the statistical model. Probability theory permits statistical projections based on incomplete knowledge. But, according to the quantum mechanical definition of "probability", before the sampling, the number is between negative infinity and positive infinity simultaneously, and the act of sampling creates the money in the wallets of the visitors. Another example is that the capture of the numbered balls by a lottery machine creates the numbers on lottery tickets.

The basis of this paradox is that *statistical theory is based on an existing deterministic reality with incomplete information; whereas, quantum measurement acts on a "probability density function" to determine a reality that did not exist before the measurement.*

Consider the simplest of systems, the "ground state wave function of the hydrogen atom". The wave function ascribes the electron an infinite number of positions and energies simultaneously including states with infinite negative kinetic energy and infinite positive potential energy. But, experiments are consistent with 13.6 eV of kinetic energy and 27.2 eV of potential energy in every case. Atomic hydrogen has been extensively studied. For example, hydrogen has long been the focus of combustion research involving kinetic and spectroscopic studies. Neutral scattering of hydrogen atoms is always observed. How can the point electron have a statistical distribution of separate positions and energies and still always give rise to neutral scattering? If position follows a probability density function, why do all of the positions give rise to the identical scattering behavior wherein the positions of the point particles (proton and electron of the incoming hydrogen atom and the point scattered electron) are random independent variables? How do all of the electric fields always identically cancel if positions are statistical? Even one neutral scattering event violates Chebyshev's inequality.

It is also nonsense to interpret the "probability" ascribed by the wave function of the electron as equivalent to that of statistical thermodynamics. The latter corresponds to a classical statistical distribution of particles such as photons or molecules over states such as energy states for a macroscopic system. It is nonsensical to assign a single particle (e.g. an electron) to a statistical distribution over many states. It is similarly nonsensical to associate a stable property such as a thermodynamic property to a single particle based on such statistics.

Quantum mechanical textbooks express the movement of the electron, and the Heisenberg uncertainty principle is an

expression of the statistical aspects of this movement. McQuarrie [8, backcover], gives the electron speed in the $n = 1$ state of hydrogen as 2.18764×10^6 m/s. Remarkably, the uncertainty in the electron speed according to the uncertainty principle is 1.4×10^7 m/s [8, p. 38] which is an order of magnitude larger than the speed. With Penning traps, it is possible to measure transitions of electrons of single ions. This case can be experimentally distinguished from statistics over equivalent transitions in many ions. Whether many or one, the transition energies are always identical within the resonant line width. So, probability has no place in describing atomic energy levels on this basis either.

According to quantum mechanics, the existence of the electron in the nucleus is the basis of spin-nuclear coupling called Fermi contact interaction [88] where $4\pi r^2 \Psi^2 \, dr$ is not zero since the Ψ^2 is not zero and the nucleus is comprised of baryons. According to the *standard model*, baryons as opposed to leptons have structure, contain more fundamental particles—namely quarks and gluons, and are not point particles. For example, the proton has an experimentally measured radius of $r_p = 1.3 \times 10^{-15}$ m. The spin-nuclear coupling energy is of the order of 10^{-24} J despite the infinite Coulombic energy of the electron when found in the nucleus (i.e. $r \rightarrow 0$ in the Schrödinger equation). This consequence of quantum mechanics is further flawed since this state is experimentally disproved. The nucleus does not contain electrons [81, p. 407]. Since the electron has no volume, based on this logic, the probability that an electron can capture a photon to form an excited state is zero. This internal inconsistency based on the description of the electron as a point particle probability wave does not arise in Mills classical theory of quantum mechanics. The spin nuclear energies are calculated by Mills in closed form based on first principles without the requirement that the electron is in the nucleus [2, pp. 98–109] and are in close agreement with the experimental results.

What does the wave function represent? It certainly does not represent the probability that a point particle will be found in a given region in space at any given instant of time if sampled. And, it has negative as well as positive values of probability which is nonsensical. This is circumvented by squaring it. But, why not take the root mean squared value or the magnitude? In the case of a wave such as an acoustic or electromagnetic wave, the energy or intensity is given as the square of the amplitude. But, when did it become correct that a probability density function (pdf) has an energy or intensity? Quantum theory confuses the concepts of a wave and a pdf that are based on totally different mathematical and physical principles. It is further meaningless to normalize such a function with the interpretation that the expectation value must be one. In addition, the postulates of the "probability" wave are internally inconsistent since negative probability density functions are often invoked to describe "antibonding orbitals" in molecules. It appears that the word "probability" as applied to quantum mechanics has nothing to do with classical probability theory.

A.5. The Schrödinger equation fails to explain scattering experiments

Scattering experiments are not consistent with an uncertainty in the radial or angular position of the electron. The Schrödinger equation interpreted as a probability wave of a point particle cannot explain that the hydrogen atom is neutral. For example, it cannot explain neutral scattering of electrons or light from hydrogen. The point particles must align perfectly; otherwise Rutherford scattering would be observed. In this case, the uncertainty principle is violated. The Born interpretation can only be valid if the speed of the electron is equal to infinity. (The electron must be in all positions weighted by the probability density function during the time of the scattering event). The correct aperture function for the Born interpretation is a Dirac delta function having a Fourier transform of a constant divided by s^2 which is equivalent to the case of the point nucleus (the Rutherford equation). The Born interpretation must be rejected because the electron velocity cannot exceed the speed of light without violating special relativity.

The elastic scattering of electrons from an atomic beam of helium atoms provides a test of the wave function solutions of the Schrōinger equation. Mills [2, pp. 19–213] gives a closed form solution of the elastic scattering of 500 eV electrons from helium atoms as the Fraunhofer pattern in the far field. Mills gives a closed form equation of the free electron [2, pp. 110–120] and a closed form equation of the helium atom [2, pp. 176–191]. This is the case of $Z = 2$, in the closed form equation that Mills has solved for all two electron atoms. The calculation is a Fourier optics type which reduces to a spherical lens calculation. The math is well known. The resultant closed form equation has no adjustable parameters. The predictions identically and continuously match the experimental scattering data [89–91]. In the case of the quantum mechanical calculation, the calculation is on a point-by-point basis without regard to internal consistency or physical laws. Furthermore, it is unstable—blows up to positive or negative infinity based on roundoff error, contains adjustable parameters, and in the words of the authors, "at smaller scattering angles, however, the Born approximation calculation fails utterly, the experimental curve rising much more steeply than the theoretical" [89].

Furthermore, QM calculations are not even internally consistent. Take the inelastic scattering of electrons from He atoms as an example. Rather than using point particles which is the norm, a single plane wave is used as the equation of all of the incident electrons [92]. And, the Schrödinger equation predicts that each of the functions that corresponds to a highly excited state electron as well as the ionized electron are sinusoidal over all space which is nonsensical, are not integrable, and can not be normalized. Thus, each is infinite [7]. The SE does not give a point or a plane wave as the function of an ionized electron.

Measurements of the resistivity of hydrogen as a function of pressure provides a test of quantum mechanics which is

similar to scattering experiments. The scalar and vector potential functions of the electrons of the hydrogen molecule are given by Mills [2, pp. 234–259] as a closed form solution of the Laplacian in elliptic coordinates with the nonradiative boundary constraint. The many solutions of the hydrogen molecule based on quantum mechanics have many flaws such as internal inconsistency, violation of fundamental laws, and use of variable parameters such as "effective nuclear charge for the proton". For example, Kolos and Wolniewicz [93,94] use a 100 terms and an effective nuclear charge of 1.072. Whereas, Kolos and Roothaan [95] use an effective nuclear charge of 1.197, and their predicted bond energy is 30% less than the experimental value. The proton charge in Mills' closed form calculation is identically equal to the experimentally measured charge of the proton, and Mills' bond energy matches the experimental energy. Recent measurements of the resistivity of hydrogen as a function of pressure confirm Mills' solution rather than quantum mechanics. The finite dimensions of the hydrogen molecule are evident in the plateau of the resistivity versus pressure curve of metallic hydrogen [96]. This is in contradiction to the predictions of quantum probability functions such as an exponential distribution in space.

A.6. It has been shown experimentally that the Heisenberg uncertainty principle has nothing to do with wave–particle duality

Feynman states [97], "It is impossible to design an apparatus to determine which hole the electron passes through, that will not at the same time disturb the electrons enough to destroy the interference pattern". If an apparatus is capable of determining which hole the electron goes through, it *cannot* be so delicate that it does not disturb the pattern in an essential way. No one has ever found (or even thought of) a away around the uncertainty principle. *so we must assume that it describes a basic characteristic of nature*".

Feynman's position has recently been over turned by an experiment by Durr et al. [98]. According to Gerhard Rempe [99], who lead the Durr et al. experimental team, "The Heisenberg uncertainty principle has nothing to do with wave–particle duality". Durr et al. report, "We show that the back action onto the atomic momentum implied by Heisenberg's position-momentum uncertainty relation cannot explain the loss of interference".

A.7. Uncertainty principle

More than 60 years after the famous debate between Niels Bohr and Albert Einstein on the nature of quantum reality, a question central to their debate—the nature of quantum interference—has resurfaced. The usual textbook explanation of wave–particle duality in terms of unavoidable "measurement disturbances" is experimentally proven incorrect by an experiment reported by Durr et al. [98]. They report on the interference fringes produced when a beam of cold atoms is diffracted by standing waves of light. Their interferometer displayed fringes of high contrast—but when they manipulated the electronic state within the atoms with a microwave field according to which path was taken, the fringes disappeared entirely. The interferometer produced a spatial distribution of electronic populations which were observed via fluorescence. The microwave field canceled the spatial distribution of electronic populations. The key to this new experiment was that although the interferences are destroyed, the initially imposed atomic momentum distribution left an envelope pattern (in which the fringes used to reside) at the detector. A careful analysis of the pattern demonstrated that it had not been measurably distorted by a momentum kick of the type invoked by Bohr, and therefore that any locally realistic momentum kicks imparted by the manipulation of the internal atomic state according to the particular path of the atom are too small to be responsible for destroying interference.

Durr et al. conclude that the "Heisenberg uncertainty relationship has nothing to do with wave–particle duality" and further conclude that the phenomenon is based on entanglement and correlation. Their interpretation of the principles of the experiment is that directional information is encoded by manipulating the internal state of an atom with a microwave field, which entangles the atom's momentum with its internal electronic state. Like all such entangled states, the constituent parts lose their separate identity. But the attachment of a distinguishable electronic label to each path means that the total electronic-plus-path wave function along one path becomes orthogonal to that along the other, and so the paths cannot interfere. By encoding information as to which path is taken within the atoms, the fringes disappear entirely. The internal labeling of paths does not even need to be read out to destroy the interferences: all you need is the option of being able to read it out.

According to Durr et al., the mere existence of information about an entity's path causes its wave nature to disappear. But, correlations are observations about relationships between quantities and do not cause physical processes to occur. The existence of information about an entity's path is a consequence of the manipulation of the momentum states of the atoms which resulted in cancellation of the interference pattern. It was not the cause of the cancellation. The cancellation is calculated by Mills [2, pp. 405–413] as the superposition of two single slit patterns as opposed to a double slit pattern which is based on determinism. In this case, an appropriate replacement terminology for "correlations are responsible for the loss of the interference pattern" is "determinism is responsible for the loss of the interference pattern".

In contrast to QM, in Mills' theory, a particle has one position and one energy in the absence of measurement. It does not have an infinite number of superimposed positions and energies simultaneously with no physical form until measurement is made. The rise-time band width relationship holds during measurement. This is a conservation statement.

Mills regards the Heisenberg uncertainty principle which is based on the probability model of fundamental particles to be wrong as well as the implicit spontaneous creation of energy and virtual particles from a perfect vacuum. The Heisenberg uncertainty principle violates first principle laws which are directly proven experimentally, and the predicted virtual particles generated from vacuum are rejected since they are not experimentally observed. Mills' theory predicts wave particle duality nature of light and particles based on first principles rather than requiring that different physics applies on the atomic scale.

A.8. The Heisenberg uncertainty principle predicts nonlocality, noncausality, spooky actions at a distance, and perpetual motion which can be shown to be experimentally incorrect

A.8.1. Flawed interpretation of the results of the Aspect experiment—there is no spooky action at a distance

Bell [100] showed that in a Gedanken experiment of Bohm [101] (a variant of that of EPR) no local hidden-variable theory can reproduce all of the statistical predictions of quantum mechanics. Thus, a paradox arises from Einstein's conviction that quantum-mechanical predictions concerning spatially separated systems are incompatible with his condition for locality unless hidden variables exist. Bell's theorem provides a decisive test of the family of local hidden-variable theories (LHVT). In a classic experiment involving measurement of coincident photons at spatially separated detectors, Aspect [102] showed that local hidden-variable theories are inconsistent with the experimental results. Although Aspect's results are touted as a triumph of the predictions of quantum mechanics, the correct coincidence rate of detection of photons emitted from a doubly excited state of calcium requires that the z component of the angular momentum is conserved on a photon pair basis. As a consequence, a paradox arises between the deterministic conservation of angular momentum and the Heisenberg uncertainty principle. The prediction derived from the quantum nature of the electromagnetic fields for a single photon is inconsistent with Aspect's results, and Bell's theorem also disproves quantum mechanics. Furthermore, the results of Aspect's experiment are predicted by Mills' theory wherein locality and causality hold. Mills derives the predicted coincidence rate based on first principles [2, pp. 511–521]. The predicted rate identically matches the observed rate.

The Aspect experiment is a test of locality and local hidden variable theories. The Aspect experiment is also a test of quantum mechanics and the HUP. In one design of the experiment, photons are incident to a beam splitter which causes each photon to be split into two that travel along opposite paths to separate detectors. The separate detectors measure the polarization of the arriving photons. By using synchronous detection, photons of a pair may be later compared. The data indicate a random pattern at each detector individually; however, when photons are matched up as pairs, an essentially perfect correlation exists. The quantum mechanical explanation is that before the photon was split its state of polarization was indeterminate. It possessed an infinite number of states in superposition. Then when one element of the pair was detected information traveled instantaneously (infinitely faster than the speed of light—otherwise known as a spooky action at a distance) to cause the other photon to have a matching polarization. In quantum mechanical terms, the states were entangled, and measurement of one photon caused the other photon's wave function to collapse into the matching state.

The correct explanation is that each photon entering the beam splitter originally had a determined state, and angular momentum was conserved on a photon-by-photon basis at the splitter. Thus, each photon of a pair had a matching polarization before it hit the detector. Locality and cause and effect hold. There is no spooky action at a distance. This experiment actually disproves quantum mechanics. It also disproves local hidden variable theories. The data of the Aspect experiment matches a classical derivation, not a quantum mechanical one.

Everyday observation demonstrates that causality and locality always hold. Bell's theorem postulates that a statistical correlation of $A(a)$ and $B(b)$ is due to information carried by and localized within each photon, and that at some time in the past the photons constituting one pair were in contact and in communication regarding this information. This is the case in many everyday experiences such as transmission, processing, and reception of signals in microelectronics devices. Locality and causality always hold. They hold on the scale of the universe also. But, according to the Big-Bang theory of quantum mechanics all photons were at one time in contact; thus, no locality or causality should be observed at all. This is nonsense. The results of the Aspect experiment support the EPR paradox that QM does not describe physical reality. There is a mistake in the derivation of the analysis of the data from Aspect's experiment [103,104].

Bell's theorem is just an inequality relationship between *arbitrary* probability density functions with certain assumptions about independence, expectation value equal to one, etc. wherein an additional probability distribution function is introduced which may represent local hidden variables or something else for that matter. And, the initial functions may correspond to quantum mechanical statistics or something else for that matter. Standard probability rules are accepted such as the probability of two independent events occurring simultaneously is the product of their independent probabilities. What is calculated and plugged into the formula for the functions and whether the substitutions are valid are the issues that determine what Bell's inequality tests when compared with data. Historically, Bell's inequality is a simple proof of statistical inequalities of expectation values of observables given that quantum statistics is

correct and that the physical system possesses "hidden variables". However, if deterministic statistics are actually calculated and quantum statistics is equivalent to deterministic statistics (e.g. detection of a wave at an inefficient detector) but possesses further statistics based on the probability nature of the theory (statistical conservation of photon angular momentum), then Bell's inequality is actually testing determinism versus quantum theory when compared to the data.

Rather than demonstrating that QM does not give us all of the information about the physical world, the data is consistent with the result that QM does not describe the physical world, and that deterministic physics does. A deterministic theory is not required to posses local hidden variables. Maxwell's equations is a deterministic theory. It does not have local hidden variables (LHV). There is no corresponding statistical distribution function. Bell's theorem is a simple proof of statistical inequalities of expectation values of observables given that "*quantum*" statistics is correct and that the physical system possesses "hidden variables" corresponding to an additional statistical distribution function. What was actually derived to explain the results of the Aspect experiment [102] was a classical calculation of the detection of an extended particle, the polarized photon, at an inefficient detector wherein determinism holds with respect to conservation of angular momentum [103,104]. Thus, the statistics defined as "*quantum*" was actually deterministic. (The derivation is given by Mills [2, pp. 511–521]). Furthermore, in actuality, quantum statistics must also possess other statistical distribution functions corresponding to the probability nature of the theory such as a statistical distribution for the z component of angular momentum which is conserved statistically as the number of photons goes to infinity. Thus, the real quantum mechanics statistics corresponds to a local hidden variable theory (LHVT) with respect to the definitions of the arbitrary probability distribution functions in Bell's inequality. Aspect recorded the expectation value of the coincidence rate at separated randomly oriented inefficient polarization analyzers for pairs of photons emitted from a doubly excited state calcium atom. The data showed a violation of Bell's inequality. This proves determinism and the real QM statistics fails the test. Furthermore, the observed coincidence count rate of Aspect [102] is equal to that predicted classically from the statistics of measurement at an inefficient detector only. The additional finite distribution function required in the case of quantum mechanics and QED results in incorrect predictions. There is no spooky action at a distance.

The Aspect experiment shows that momentum in conserved on a photon-by-photon basis, not statistically as predicted by the HUP. Similar experiments regarding tests of entanglement predicted by the HUP are shown to be consistent with first principle predictions and reveal flaws in the interpretations based on the HUP. The HUP implies nonlocality, noncausality, and spooky actions at a distance which can be shown to be experimentally incorrect.

A.8.2. Flawed interpretation of the results on a single $^9Be^+$ ion in a trap in a continuous Stern–Gerlach experiment—an ion cannot be at two places at the same time

There is a mistake in the analysis of the data from Monroe et al. [105]. Their interpretation that the same beryllium ion was observed to be at widely separated points at the same time is absolute nonsense. Their experimental results show that locality and causality hold [2, pp. 521–536].

A report in New York Times [106] entitled "Physicists put atom in 2 places at once" states, "a team of physicists has proved that an entire atom can simultaneously exist in two widely separated places". The article further states, "In the quantum "microscale" world, objects can tunnel magically through impenetrable barriers. A single object can exist in a multiplicity of forms and places. In principle, two quantum-mechanically "entangled" objects can respond instantly to each other's experiences, even when the two objects are at the opposite ends of the universe". Experimentally, interference patterns were observed by Monroe et al. [105] for a single $^9Be^+$ ion in a trap in a continuous Stern–Gerlach experiment. Monroe's interpretation of the experimental observation was that the ion wave function interfered with itself wherein the ion was at two separate places at the same time corresponding to a wave function state called a "Schrödinger cat" state [105–107]. According to Monroe et al.,

> A "Schrödinger cat"-like state of matter was generated at the single atom level. A trapped $^9Be^+$ ion was laser-cooled to the zero-point energy and then prepared in a superposition of spatially separated coherent oscillator states. This state was created by application of a sequence of laser pulses, which entangles internal (electronic) and external (motional) states of the ion. The "Schrödinger cat" superposition was verified by detection of the quantum mechanical interference between the localized wave packets. This mesoscopic system may provide insight into the fuzzy boundary between the classical and quantum worlds by allowing controlled studies of quantum measurement and quantum decoherence.

The "Schrödinger cat" state analysis relies on the postulate that the *Pauli exclusion principle* applies to Rabi states wherein a rotation of the magnetic moment of the unpaired electron of an RF- trapped $^9Be^+$ ion is represented by a linear combination of spin $1/2$ ($|\uparrow\rangle_i$) and spin $-1/2$ ($|\downarrow\rangle_i$) states. Three steps of rotation of the spin magnetic moment by a time harmonic field provided by pairs of copropagating off-resonant laser beams which drove two-photon-stimulated Raman magnetic resonance transitions were each separated by displacement laser pulses which excited a resonant translational harmonic oscillator level of the trapped ion by coupling only with the $|\uparrow\rangle_i$ state. According to Monroe, "this selectivity of the displacement force provides quantum entanglement of the internal state with the external motional

state. Although the motional state can be thought of as nearly classical, its entanglement with the internal atomic quantum levels precludes any type of semiclassical analysis". The interference was detected by exciting a fluorescent transition which only appreciatively coupled to the $|\downarrow\rangle_i$ state. Thus, the fluorescence reading was proportional to the probability $P_\downarrow$ the ion was in state $|\downarrow\rangle_i$. The "Schrödinger cat" superposition was supposedly verified by detection of the quantum mechanical interference between the localized wave packets.

However, the interference arises not from the existence of the ion at two places at once. The positively charged ion was excited to a time harmonic translational energy state, and the spin quantization axis was defined by an applied 0.20 mT magnetostatic field at an angle of $\pi/4$ with respect to the x-axis of the RF-trap. The frequency of the energy to "flip" the spin state was equivalent to the projection of that of the translational harmonic oscillator onto the spin axis

$$\frac{\omega_x}{2\pi}\cos^2\frac{\pi}{4} = (11.2\ \text{MHz})(0.5) = 5.605\ \text{MHz} = \frac{\Delta E_{\text{mag}}^{\text{spin}}}{h} \tag{A.2}$$

given by Eqs. (37.45)–(37.48) of Mills [2, pp. 521–566]. Thus, interference occurred between the Stern–Gerlach transition and the synchrotron radiation corresponding to the charged harmonic oscillator. Since the displacement beams affected only motion correlated with the $|\uparrow\rangle_i$ state, a rotation of the magnetic moment such that $\delta \neq 0$ with application of the displacement beams gives rise to a phase shift of the interference pattern. The closed form calculation is given in Mills [2, pp. 521–536].

A.8.3. Flawed interpretation of the results of experiments on a small SQUID coupled to a biased large superconducting current loop—a superconducting current cannot flow in opposite directions at the same time

There is a mistake in the analysis of the data from Friedman et al. [108]. Their interpretation that a superconducting current loop can exist as a superposition of contradictory states at the same time is absolute nonsense. It is shown by Mills [2] that their experimental results are consistent with locality and causality.

A recent report in The New York Times [109] entitled "Here, There and Everywhere: A Quantum State of Mind" states, "Physicists at Delft University of Technology have put a 5-micrometer-wide loop of superconducting wire into a "quantum superposition" of two contradictory possibilities: in one, the current flows clockwise; in the other, current flows counterclockwise". The article further states, "In the realm of atoms and smaller particles, objects exist not so much as objects as mists of possibilities being here, there and everywhere at the same time—and then someone looks and the possibilities suddenly collapse into definite locations". The experiment was a simplified version of the concept of Schrödinger's cat. In 1935, Schrödinger [110] attempted to demonstrate the limitations of quantum mechanics using a thought experiment in which a cat is put in a quantum superposition of alive and dead states.

Instead of a cat, Friedman et al. [108] used a small square loop of superconducting wire linked to a SQUID (Superconducting Quantum Interference Device). A SQUID comprises a superconducting loop with a Josephson junction, a weak link that causes magnetic flux to be linked in integer units of the magnetic flux quantum. When the loop is placed in an external magnetic field, the loop spontaneously sets up an electrical current to cancel the field or generate an additional magnetic field, adjusting the magnetic field to a unit of the magnetic flux quantum, one of the allowed values. In the experiment of Friedman et al., the loop was placed in a magnetic field equal to one half of the first allowed value, a magnetic flux quantum. Thus, the loop could set up either a current to raise the field strength to the first allowed value, or with equal probability, a current of equal magnitude flowing in the opposite direction to cancel out the external field. A pulse of microwaves was applied at the frequency to cause a transition of the magnetic moment of the current loop as an entirety. The absorption of microwaves caused the magnetic state of the SQUID to change and the current to reverse its direction.

Experimentally, a measurement always gave one of the two possible answers, clockwise or counterclockwise, never a zero cancellation. A difference in energy at which the flip transition occurred between the two possibilities was detected by a group led by J. Lukens and J. Friedman at the State University of New York (SUNY). A simple explanation was that the microwaves simply flipped the current direction which had an energy bias in one direction versus the opposite based on the corresponding presence or absence of a magnetic flux quantum within the SQUID. Rather, they interpreted the results as experimental evidence that a SQUID can be put into a superposition of two magnetic flux states: one corresponding to a few microamperes of current flowing clockwise and the other corresponding to the same amount of current flowing anticlockwise. "Just as the cat is neither alive nor dead but a ghostly mix of the two possibilities, the current flows neither clockwise or counterclockwise, but is a mix of the two possibilities [109]". According to Friedman, "we can have two of these macroscopically well-defined states at the same time. Which is something of an affront to our classical intuitions about the world [109]".

Current running in both directions simultaneously is nonsensical. Current is a vector and must have only one direction. The energy difference observed by Friedman et al. can be explained *classically*. The experimental apparatus comprised a small SQUID coupled to a large current loop. A second SQUID magnetometer read the flux state of the first sample SQUID. The energy difference was not due to superposition of flux states. Rather, it was due to the nature of the electron which carries the superconducing current and links flux in units of the magnetic flux quantum. Consequently, the sample SQUID linked zero or one magnetic flux

quantum. When excited by electromagnetic radiation of a resonant frequency, individual electrons undergo a spin-flip or Stern–Gerlach transition corresponding to a reversal of the electron magnetic moment, angular moment, and current. The Stern–Gerlach transition energies of electrons superimpose. The energy difference observed by Friedman et al. matches the energy corresponding to the flux linkage of the magnetic flux quantum by the ensemble of superconducting electrons in their entirety with a reversal of the corresponding macroscopic current. The linkage was caused by high power microwave excitation of a Stern–Gerlach transition of the magnetically biased loop which caused a concomitant change in the flux state of the separately magnetically biased sample SQUID. In this case, the microwave frequency was kept constant, and the bias flux of the loop was scanned at a fixed magnetic bias of the sample SQUID until the resonance with the superposition of the Stern–Gerlach transitions of the superconducting electrons in their entirety was achieved.

A.8.4. Flawed prediction of perpetual motion by the Heisenberg uncertainty principle

Another consequence of HUP wherein entanglement of states is implicit is the prediction of perpetual motion. Schewe and Stein report on the work of Allahverdyan and Nieuwenhuizen [111]:

Armen Allahverdyan of, CEA Saclay (France)/University of Amsterdam (Netherlands)/Yerevan Physics Institute (Armenia), aarmen@spht.saclay.cea.fr, and Theo Nieuwenhuizen of the University of Amsterdam (nieuwenh @wins.uva.nl, 011-31-20-525- 6332) [112] suggest that a quantum particle (such as an electron) interacting strongly with a reservoir of particles may violate the Clausius inequality—one formulation of the second law of thermodynamics, which states that it is impossible to do work without losing heat. What the researchers term "appalling behavior" can be traced to the quantum mechanical property of entanglement, in which a quantum particle (such as an electron) is so strongly interlinked with another particle or group of particles that the resulting behavior cannot be treated by standard thermodynamic approaches. In this paper, the Amsterdam scientists study the entanglement of a particle with a "quantum thermal bath", a reservoir of particles with which the first particle can exchange energy and momentum. According to the researchers, entanglement prevents the quantum bath from observing the normal requirements for a heat bath. Therefore, thermodynamics simply cannot say anything useful about the system.

Standard thermodynamics dictates that the bath be in thermal equilibrium and not interact strongly with an external object. To the contrary, the bath strongly interacts with something external to it (the entangled particle) and it cannot reach equilibrium, since it constantly exchanges energy and momentum with the particle. At low temperatures where entanglement could be easily preserved, the researchers state that this system can apparently violate the Clausius inequality—in which the heat gained by the particle must be less than or equal to the temperature multiplied by the change in its entropy (or disorder). Near absolute zero temperatures, a situation which would ordinarily require the particle to lose heat, the researchers show that the particle could gain heat, by the Clausius relation. According to this scenario, applying a cyclic parameter such a periodically varying external magnetic field can cause the entangled particle to extract work from the bath—something forbidden in a classical system. *Further, the researchers say that this phenomenon could be said to constitute a perpetual motion machine of the second kind.*

A.9. The Postulated Schrödinger equation does not explain the stability of the hydrogen atom

QM theory does not say why an atom radiates. Quantum states of QM refer to energy levels of probability waves. From these, emission and absorption of radiation is inferred. But QM does not explain why it is emitted or absorbed or why certain states are stable. For example, the Schrödinger equation (SE) was *postulated* in 1926. It does not explain the stability of the hydrogen atom. To say that the atom obeys the SE is nonsensical. Consider the hydrogen atom without regard to the mathematical formula called the SE. Mathematics does not determine physics. It only models physics. The SE is not based on directly testable physical laws such as Maxwell's equations. It only gives correlations, and is in fact inconsistent with physical laws.

As a historical note:

> [My father] said, "I understand that they say that light is emitted from an atom when it goes from one state to another, from an excited state to a state of lower energy."
>
> I said, "That's right."
>
> "And light is kind of a particle, a photon, I think they call it."
>
> "Yes."
>
> "So if the photon comes out of the atom when it goes from the excited to the lower state, the photon must have been in the atom in the excited state."
>
> I said, "Well no."
>
> He said, "Well, how do you look at it so you can think of a particle photon coming out without it having been there in the excited state?"
>
> I thought a few minutes, and I said, "I'm sorry; I don't know. I can't explain it to you."
>
> -Richard P. Feynman, The Physics Teacher (September 1969).

As shown in "Schrödinger states below $n = 1$" section, the definition of the "ground state" is mathematically purely arbitrary. It is always experimentally observed that the hydrogen atom does not spontaneously emit light once it has achieved an energy level of 13.6 eV. Thus, it is taught in textbooks that atomic hydrogen cannot go below the ground

state. But, atomic hydrogen having an experimental ground state of 13.6 eV can only exist in a vacuum or in isolation, and atomic hydrogen cannot go below this ground state only when it is in isolation. Atomic hydrogen is extremely reactive, and there is no known composition of matter containing hydrogen in the ground state of 13.6 eV.

Since the Schrödinger equation offers no foundation for the stability of isolated atomic hydrogen, Feynman attempted to find a basis for the definition of the "ground state" in the Heisenberg uncertainty principle [97, p. 2–6]. Feynman's based his derivation on the determination of the momentum as $p \approx \hbar/a$ from the uncertainty principle wherein he argues, "We need not trust our answer to within factors like 2, π, etc. We have not even defined a very precisely". The kinetic energy follows classically from the momentum, and the electrostatic energy is given classically to give the total energy as

$$E = \hbar^2/2ma^2 - e^2/a. \tag{A.3}$$

Feynman determined the minimum energy in order to solve for the radius of the hydrogen atom.

$$dE/da = -\hbar^2/ma^3 + e^2/a^2 = 0. \tag{A.4}$$

The result is exactly the Bohr radius.

The uncertainty principle [8, pp. 135–140] is

$$\sigma_x \sigma_p \geq \hbar/2, \tag{A.5}$$

where σ_x and σ_p are given by

$$\sigma_x^2 = \int \psi^*(\hat{X} - \langle x \rangle)^2 \psi \, dx, \tag{A.6}$$

$$\sigma_p^2 = \int \psi^*(\hat{P} - \langle p \rangle)^2 \psi \, dx. \tag{A.7}$$

The definition of the momentum operator in a *one-dimensional* system is [8, pp. 135–140]

$$\hat{P}_x = -i\hbar \frac{d}{dx} \tag{A.8}$$

and the position operator is

$$\hat{X} = x \quad \text{(multiply by } x\text{)}. \tag{A.9}$$

Based on the uncertainty principle, Feynman's derivation of the Bohr radius is flawed on the basis of at least five points:

(1) The uncertainty principle gives a lower limit to the product of the *uncertainty* in the momentum and the position—*not* the momentum and the position. The momentum or position could be arbitrarily larger or smaller than its uncertainty. For example, quantum mechanical textbooks express the movement of the electron, and the Heisenberg uncertainty principle is an expression of the statistical aspects of this movement. McQuarrie [15], gives the electron speed in the $n = 1$ state of hydrogen as 2.18764×10^6 m/s. Remarkably, the uncertainty in the electron speed according to the uncertainty principle is 1.4×10^7 m/s [8, p. 38] which is an order of magnitude larger than the speed.

(2) Feynman's derivation of the Bohr radius is internally inconsistent since the kinetic and electrostatic energies were derived *classically*; whereas, quantum mechanics and the uncertainty principle are not consistent with classical mechanics.

(3) Feynman's derivation of the Bohr radius is internally inconsistent since the uncertainty principle requires *uncertainty* in the position and momentum. Yet, Eqs. (2.10) and (2.11) of Feynman (Eqs. (A.2)–(A.4)) can be solved to give an *exact* rather than a most probable electron position, momentum, and energy.

(4) Feynman's derivation of the Bohr radius is flawed since Eq. (2.11) of Feynman (Eq. (A.2)) is nothing more than the Bohr force balance equation given by McQuarrie [8, pp. 22–26] and also derived by Mills [7]. Thus, this approach fails at explaining the stability of the 13.6 eV state beyond an arbitrary definition wherein "We need not trust our answer to within factors like 2, π, etc. [97, p. 2–6]".

(5) The faulty logic is compounded by the fact that the uncertainty principle is founded on the definition of the momentum operator given by Eq. (A.8) and the position operator given by Eq. (A.9). Thus, the uncertainty principle is based on the postulated Schrödinger equation and its associated postulates and descriptions of particles as probability waves. *It is not based on physics.* In fact, it is nonsensical in many physical tests such as scattering of electrons from neutral atoms, confining electrons to atoms, confining electrons to atoms in excited states wherein a photon causing a transition carries $\hbar$ of angular momentum, and the cosmological consequences of the uncertainty principle as described previously. Also, it is disproved experimentally that it provides a basis for the wave–particle duality nature of light and particles; even though, the opposite is widely touted as discussed in the "It has been shown experimentally that the Heisenberg uncertainty principle has nothing to do with wave-particle duality" section.

According to the generally accepted Born interpretation of the meaning of the wave function, the probability of finding the electron between r, θ, ϕ and $r + dr, \theta + d\theta, \phi + d\phi$ is given by Eq. (A.1). The electron *is viewed as a discrete particle* that moves here and there (from $r = 0$ to $r = \infty$), and $\Psi\Psi^*$ gives the time average of this *motion*. The Schrödinger equation possesses terms corresponding to the electron radial and angular kinetic energy which sum with the potential energy to give the total energy. These are necessary conditions for an electron bound by a central field [10]. Herman Haus derived a test of radiation based on Maxwell's equations [15]. Applying Haus's theorem to the point particle that must have radial kinetic energy demonstrates that the Schrödinger solution for the $n = 1$ state of hydrogen is radiative; thus, it violates Maxwell's equations. Since none is observed for the $n = 1$ state, QM is inconsistent with observation. The derivation is shown in the "Schrödinger wave

functions in violation of Maxwell's equations" section of Mills [2, pp. 487–489].

In contrast, the classical theory of Mills is derived from Maxwell's equation with the constraint that the $n = 1$ state is nonradiative. This approach leads to the prediction of stable states below the traditional $n = 1$ state. Corresponding states are confirmed by the data on the free electrons in superfluid helium.

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